

## Extra Review Questions- Functions and Inverses

1. State whether odd, even, 1-1 and find the domain, range.

a)  $f(x) = x - \sqrt{x}$     b)  $g(x) = \frac{x^2 + 1}{x^2 + 4}$     c)  $h(x) = \frac{x}{|x|}$     d)  $y = 1 - \sqrt{x^2 - 3x - 4}$

2. Find the inverse and state whether the inverse is a function.

a)  $y = \sin^{-1}(x + 1) + \frac{\pi}{4}$     b)  $y = \frac{x^2 - 1}{x^2 - 4}$

3. If  $f(x) = \frac{2}{\pi}x + \sin x$ , where  $0 \leq x \leq \frac{\pi}{2}$  and  $g(x) = f^{-1}(x)$ , find:

a)  $g(3)$

b)  $g'(3)$

c) Explain why  $f$  is 1-1.

4. If  $h(x) = 1 - \frac{1}{x}$ , find:

a)  $h(2)$     b)  $h(h(2))$     c)  $h(h(h(2)))$     d) prove that  $h(h(h(x))) = x$

5. If  $h(x) = 1 - \frac{1}{x}$  and  $g(x) = 2x - 2$ , find a)  $h^{-1}, g^{-1}$  b)  $g \circ h(x)$     c)  $h^{-1} \circ g^{-1}(x)$

d) comment on what your answers to b) c) suggests about order in composing inverses.

6. If  $f(x)$  is an even function and  $g(x)$  is an odd function, then comment as to whether the following is odd or even or neither:

a)  $h(x) = f(x) \times g(x)$     b)  $p(x) = f(x) + g(x)$     c)  $c(x) = f \circ g(x)$     d)  $n(x) = g \circ f(x)$

e)  $h'(x)$

7. Let  $f$  and  $g$  be differentiable functions such that

$$f(1) = 2 \quad f'(1) = 3 \quad f'(2) = -4$$

$$g(1) = 2 \quad g'(1) = -3 \quad g'(2) = 5$$

If  $h(x) = f(g(x))$ , then  $h'(1) =$

(A) -9      (B) -4      (C) 0      (D) 12      (E) 15

Answers: 1 a) neither even nor odd, not 1-1, domain  $x \geq 0$ , range  $y \geq \frac{-1}{4}$

b) even, not 1-1, domain all real x, range  $\frac{1}{4} \leq y < 1$

c) odd, not 1-1, domain  $x \neq 0$ , range  $y = 1, -1$

d) neither even nor odd, yes 1-1, domain  $x \geq 4$  or  $x \leq -1$ , range  $y \leq 1$ .

2. a) yes,  $y = \sin(x - \frac{\pi}{4}) - 1$  b) no,  $y = \pm \sqrt{\frac{-3}{x-1} + 4}$  3. a)  $\frac{\pi}{2}$  b)  $\frac{\pi}{2}$

c)  $f(x)$  increases in  $\left[0, \frac{\pi}{2}\right]$  4. a)  $\frac{1}{2}$  b) -1 c) 2 d) use algebra

5. a)  $h^{-1}(x) = \frac{1}{1-x}$ ,  $g^{-1}(x) = \frac{x+2}{2}$  b)  $\frac{-2}{x}$  c)  $\frac{-2}{x}$  d)  $g \circ h(x) = h^{-1} \circ g^{-1}(x)$

6. a) odd b) neither c) even d) odd e) odd 7. D