ExtraReview: Limits, Continuity, First Principles

1. Find the value(s) of a, b such that f(x) as defined below will be differentiable at x=1.

$f(x) = ax^3 + bx \text{ for } x > 1$

$f(x) = 3x + 2 \text{ for } x \le 1$

2. Evaluate the following limits algebraically:

a) $\lim_{x\to 16} \frac{4-\sqrt{x}}{x-16}$ b) $\lim_{x\to 3} \frac{1-\frac{9}{x^2}}{x-3}$ c) $\lim_{x\to\infty} \frac{3x^3 - x}{-3x^3 + 10x + 1}$ d) $\lim_{x\to-\infty} \frac{\sqrt{9x^2-4}}{6x+2}$

3. Find the following limits using a calculator:

a) $\lim_{x\to+\infty} \sqrt{x^2+6x+10} - \sqrt{x^2-4x}$ (try this without a calculator) b) $\lim_{x\to 1} \frac{2^{1/x}-2}{x-1}$

c) $\lim_{x\to 0^-} \frac{2^{1/x}-2}{x-1}$ d) $\lim_{x\to 0^+} \frac{2^{1/x}-2}{x-1}$ e) $\lim_{x\to 0} \frac{2^{1/x}-2}{x-1}$ f) Is the function

$f(x) = \frac{2^{1/x}-2}{x-1}$ continuous at either x=0 or at x= 1?

4. Draw a graph satisfying the following properties:

$\lim_{x\to\infty} f(x) = 2, \lim_{x\to-\infty} f(x) = -2, \lim_{x\to 2^+} f(x) = 4, \lim_{x\to 2^-} f(x) = 2, \lim_{x\to 1} f(x) = 0,$

but

$f(1) = -1$

5. Use l'Hopital's rule or the first principles definition of the derivative to find:

a) $\lim_{x\to 32} \frac{5x^{\frac{1}{5}} - 10}{\sqrt[3]{x+32} - 4}$ b) $\lim_{h\to 0} \frac{(1+h)^{100} - 1}{h}$

6. a) In the interval [0,4], find the value(s) of x such that the average slope equals the instantaneous slope for the function $f(x) = 2\sqrt{x^2+9}$

   b) For the same function, find the value of the constant k such that in the interval [0,k], the average slope equals the instantaneous slope at x=4

7. Find the derivative of $f(x) = \frac{3}{\sqrt{2x+1}}$ by first principles

Answers

1. a=-1, b=6
2. a) –1/8, b) 2/3 c) -1 d) -1/2
3. a) 5 b) about –1.39 c) 2 d) $-\infty$ e) does not exist f) no
4. lots of possibilities
5. a) 3 b) 100

6. a) $x = \sqrt{3}$ b) exact answer is $k = \frac{40}{3}$

7. $f'(x) = \frac{-3}{\sqrt{(2x+1)^3}}$