Day 8 Calculus Trigonometry Notes:

Derivative Rules: Make sure you can prove each of these using first principles and identities. Also, be aware of any restrictions in the domain and range when working with the inverse trigonometric functions.

$f(x)$	$f'(x)$
$\sin x$ --------	$\cos x$
$\cos x$ -------	$(-\sin x)$
$\tan x$ --------	$\sec^2 x$
$\cot x$ --------	$(-\csc^2 x)$
$\sec x$ --------	$\sec x \tan x$
$\csc x$ --------	$(-\csc x \cot x)$

Inverse Trig Functions

$\sin^{-1} x$ --------	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$ --------	$\left(\dfrac{-1}{\sqrt{1-x^2}}\right)$
$\tan^{-1} x$ --------	$\dfrac{1}{1+x^2}$

Important Limit

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

Prove the derivative of sin x by first principles. Use trigonometric identities and the rules of differentiation to prove each of the other results. (See text pages 249-258 if you are having difficulty with this)

Examples:

1. Find $\frac{dy}{dx}$ for each of the following.

a) $y=\frac{1}{3}\sin 3x-\frac{1}{3}\cos x^3$

b) $y=\sec(6x)\tan(6x)$

c) $\cot(x+2y)=\cos^2 y-\sec x+1$

d) $y=(4+16x^2)\tan^{-1}(2x)$

2. Find the equation of the tangent to $y=-2\cos(3x)-x+1$ at x = 0

3. Find: $\lim_{x\to\pi}\frac{\tan(\pi-x)}{x-\pi}$

4. Find the equation of the tangent to $\sin x \cos y = \tan(x - y)$ at the point $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$.

5.a) Find the smallest positive value of x at which the graphs of

$y = -\sin x$ and $y = 4\sin x - \frac{5\sqrt{3}}{2}$ intersect.

b) Show that at this point of intersection the tangents to each graph are perpendicular to one another.

6. Consider $y = \sin^2 x + \cos 2x$ in the interval $0 \le x \le 2\pi$. Find the intercepts and the maximum/minimum values of the function algebraically.
Sketch the function.

7. In a 100 m race, the position of the Canadian runner after seconds is given by the formula where the position is in metres.

During the same race, the position of the American runner is given by the formula .

a) Graph the two equations for and explain in words what transpired during the race.

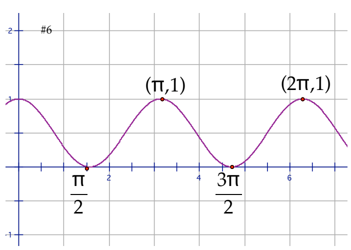
b) Find the maximum lead either runner had during the race. At what time did this take place?

Answers:

1. a) $\cos 3x + x^2 \sin x^3$ b) $6\sec 6x\left[\tan^2 6x + \sec^2 6x\right]$

c) $\dfrac{\sec x \tan x - \csc(x+2y)\cot(x+2y)}{2\csc(x+2y) - \sin y \cos y}$ d) $32x\tan^{-1}(2x) + 8$

2. y=-x-1 3. -1 4. y=x 5. a) $\frac{\pi}{3}$ b) $\because$ when $x = \frac{\pi}{3}. m_1 = \frac{-1}{2}, m_2 = 2 \Rightarrow$ perpendicular



7 a) At first, the American runner runs faster than the Canadian. However, in the end, the Canadian runner catches up and ties the American runner. They each complete the race in 10 seconds.

b) The maximum lead, by the American runner, is 25 metres after $6\frac{2}{3}$ s

Trigonometric Derivatives- Problems

1. $\lim_{x \to \pi/4} \frac{\tan(x) - 1}{x - \frac{\pi}{4}} =$

a) -1 b) 0 c) 1 d) 2 e) –2 (Answer: d)

2. $\lim_{h \to 0} \frac{\sec\left(\frac{4\pi}{3} + h\right) - \sec(\frac{4\pi}{3})}{h} =$ (Answer: c)

a) $\frac{-1}{2}$ b) $\frac{1}{2\sqrt{3}}$ c) $-2\sqrt{3}$ d) -2 e) 1

3. The height of a bouncing object at time t seconds is given by the formula $h(t) = 96\sin(\frac{\pi}{12}t) - 4\pi t$, where h is in metres. Find the times at which the velocity equals 0 in the first 25 seconds of motion. (Answers: 4,20)

4. Find the values of x for which the graphs of y= $-\sqrt{3}$ sinx and y= cosx have the same slope in the interval $0 \le x \le 2\pi$. (Answer: $x = \frac{\pi}{3}, \frac{5\pi}{3}$)

5. Find the equation of the tangent to $\sin 3x + \cos y = \tan(x + 3y) + 1$ at the point (0,0). (Answer: $y = \frac{2}{3}x$)

6 a)Find the smallest positive x value at which the graphs of y=cosecx and y=2cosx Intersect.

b) Find the slopes of each at the point of intersection.

c) Comment on what you found in part b)

Answer: a) $\frac{\pi}{4}$ b) slope of each= $-\sqrt{2}$ c) they are tangent to one another

7. The graph of the function f(x) defined below is continuous and differentiable everywhere. Find the value of the constants k and c.

$f(x) = ax^2 + bx$ for $x \le 1$

$f(x) = \cos(\pi x)$ for $x > 1$

Answer: a=1, b=-2

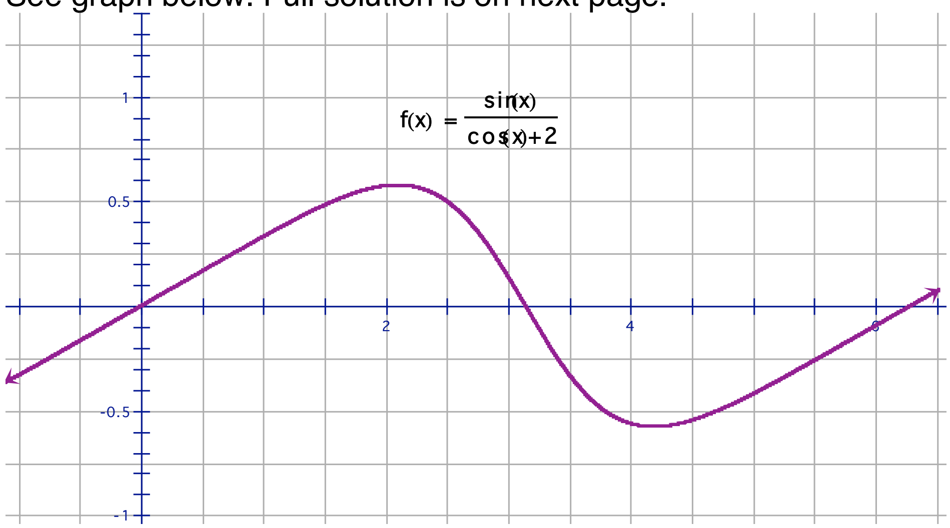
8. For the graph of $y = \dfrac{\sin x}{\cos x + 2}$ over the interval $[0, 2\pi]$ find:

a) x and y intercepts

b) relative maximum/minimum values

c) any points of inflection

See graph below. Full solution is on next page.



For x intercepts, let y=0 $\Rightarrow x = 0, \pi, 2\pi$

$$\frac{dy}{dx} = \frac{(\cos x + 2)(\cos x) - \sin x(-\sin x)}{(\cos x + 2)^2}$$

$$= \frac{\cos^2 x + 2\cos x + \sin^2 x}{(\cos x + 2)^2}$$

$$= \frac{1 + 2\cos x}{(\cos x + 2)^2}$$

Let $\frac{dy}{dx} = 0 \Rightarrow \cos x = \frac{-1}{2} \Rightarrow x = \frac{2\pi}{3}, \frac{4\pi}{3}$

at $x = \frac{2\pi}{3}$, y=$\frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{\sqrt{3}}{3}$

at $x = \frac{4\pi}{3}$, y=$\frac{-\sqrt{3}}{3}$

Note that there are no vertical or horizontal asymptotes

$$\frac{d^2y}{dx^2} = \frac{(\cos x + 2)^2(-2\sin x) - 2(\cos x + 2)(-\sin x)(1 + 2\cos x)}{(\cos x + 2)^4}$$

$$= \frac{-2\sin x(\cos x + 2)[\cos x + 2 - (1 + 2\cos x)]}{(\cos x + 2)^4}$$

$$= \frac{-2\sin x(-\cos x + 1)}{(\cos x + 2)^3}$$

For pts of inflection, let $-2\sin x(-\cos x + 1) = 0$

$\therefore \sin x = 0$ or $\cos x = 1 \Rightarrow x = \pm 2\pi, \pm\pi, 0$

$(\pm 2\pi, 0)$ are endpoints of the interval.

$\therefore$ each x-intercept is a point of inflection.