

Extra Review- Derivatives of Logarithmic, Exponential Functions

1. Find the equation of the tangent to $y = 2^{2x} - (\ln 4)x + 2\ln(x+1)$ when $x=0$.
(Answer: $y=2x+1$)

2. a) Show that the graph of $y=x - k\ln x$, where k is a positive constant, has a minimum value when $x=k$.

b) Using this fact, find the value of k such that the minimum value is 0.
(Answer: $k=e$, since k is positive)

3. Find the equation of the tangent to the graph of $y = e^x$ which passes through the origin. (Answer: $y=ex$)

4. Find the maximum value of $y = \frac{\ln x}{x^2}$.

(Answer: $\frac{1}{2e}$)

5. . The equation of the tangent to $\ln(x+y) - e^{-y} = -x$ at the point $(1,0)$ is :

a) $y=x$ b) $y = 2x - 2$ c) $y = x - 1$ d) $y = -x + 1$ e) $y = -2x + 2$

(Answer: d)

6. The tangent line to the graph of $y = e^{2-x}$ at the point $P(1,e)$ intersects both the x and the y axes. Find the area of the triangle formed by this tangent and the coordinate axes.

(Answer: $2e$ square units)

7. The velocity of a person released from a flying object (with a parachute) is given by the formula $v(t) = -200e^{-0.1t} + 25e^{0.2t}$, where t is time in seconds and velocity is measured in metres/second. A negative velocity means that the parachute is moving towards the ground.

a) Find the initial velocity (when the person was released). Answer: -175 m/s

b) If the person lands just when his velocity is 0, find the exact time it takes for this to take place. (note: at this instant, the velocity v and height h are both 0)

Answer: $t = \frac{\ln 8}{3}$

c) Find the formula for the height of the person in the parachute at time t .

Answer: $h(t) = 2000e^{-0.1t} + 125e^{0.2t} - 1500$

d) Use this information to determine the height at which the person was released.

Answer: 625 m

8. The position of a point on a number line is given by the formula
 $s(t) = 2(t - 1)^2 + \ln(4t + 1)$ where $t \geq 0$. (t in seconds, s in metres)

- Find: a) the velocity of the particle at $t = 1$ second (Answer: 4/5)
b) the times at which the particle is stopped (justify using algebra)
(Answer: at $\frac{3}{4}$ seconds)
c) the total distance travelled in the first 5 seconds

(Answer: use a calculator to find $\left|s(5) - s\left(\frac{3}{4}\right)\right| + \left|s\left(\frac{3}{4}\right) - s(0)\right|$)