

**AP Calculus Test: Intro to Derivatives**

**Name:** \_\_\_\_\_

**Total Marks=**  $\frac{\quad}{50}$

**Section A- True or false page (one mark each)**

$\frac{\quad}{6}$

**Simply put T (for true) F (for false) in blank on the right of each statement.**

For any function  $y= f(x)$ :

- A If  $f'(a) = 0$  and  $f''(a) = 0$ , then  $f(x)$  has a relative maximum at  $x=a$ . \_\_\_\_\_
- B An inflection point occurs where  $f'(x)$  has a relative maximum \_\_\_\_\_
- C An inflection point occurs where  $f(x)$  has a relative maximum \_\_\_\_\_
- D  $f(x)$  is decreasing at  $x= c$  if  $f'(x) < 0$  at  $x=c$  \_\_\_\_\_
- E If  $f'(x)$  is increasing at  $x=b$  then  $f(x)$  is concave up at  $x=b$  \_\_\_\_\_
- F  $f(x) = x^3 + x$  is an even function \_\_\_\_\_
- G If  $f(x) = x^3 + x$ , then  $f'(x)$  is an even function. \_\_\_\_\_

**Section B- Multiple Choice-** Show working in this section for questions worth **more** than one mark- Calculators may be used, but are not needed.

$\frac{\quad}{13}$

1. If  $f'(x) = (x - 1)^3 (2x - 5)^4$ , then, at  $x = \frac{5}{2}$  graph of  $f(x)$  has: (2 marks)

- a) a relative maximum
- b) a relative minimum
- c) an x intercept
- d) a change in concavity
- e) none of these

2. If  $h(x) = f(f(x))$  and it is known that (3 marks)  
 $f(4) = 2, f(2) = 6, f'(2) = -3, f'(4) = -1$ ,  
Then the value of  $h'(4)$  is

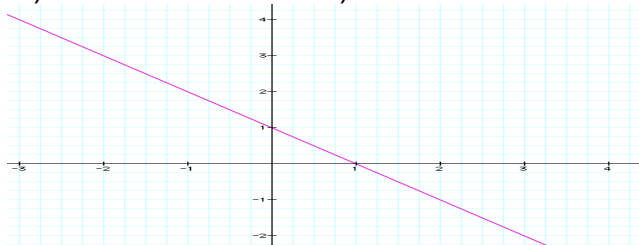
- a) 3
- b) -3
- c) 2
- d) -2
- e) -6

3. If  $y = \frac{u+1}{u^2}$  and  $u = \sqrt{2x+1} - x^2$ , then  $\frac{dy}{dx}$  when  $x=0$  is : (4 marks)
- a) 3    b) -3    c)  $\frac{3}{2}$     d)  $\frac{-3}{2}$     e) -6

4. The equation of the tangent to  $y = (x-1)^5$  at its **y** intercept is : ( 3 marks)
- a)  $y = x + 1$     b)  $y = 0$     c)  $y = 5x - 1$     d)  $y = -5x - 1$     e)  $y = x - 1$

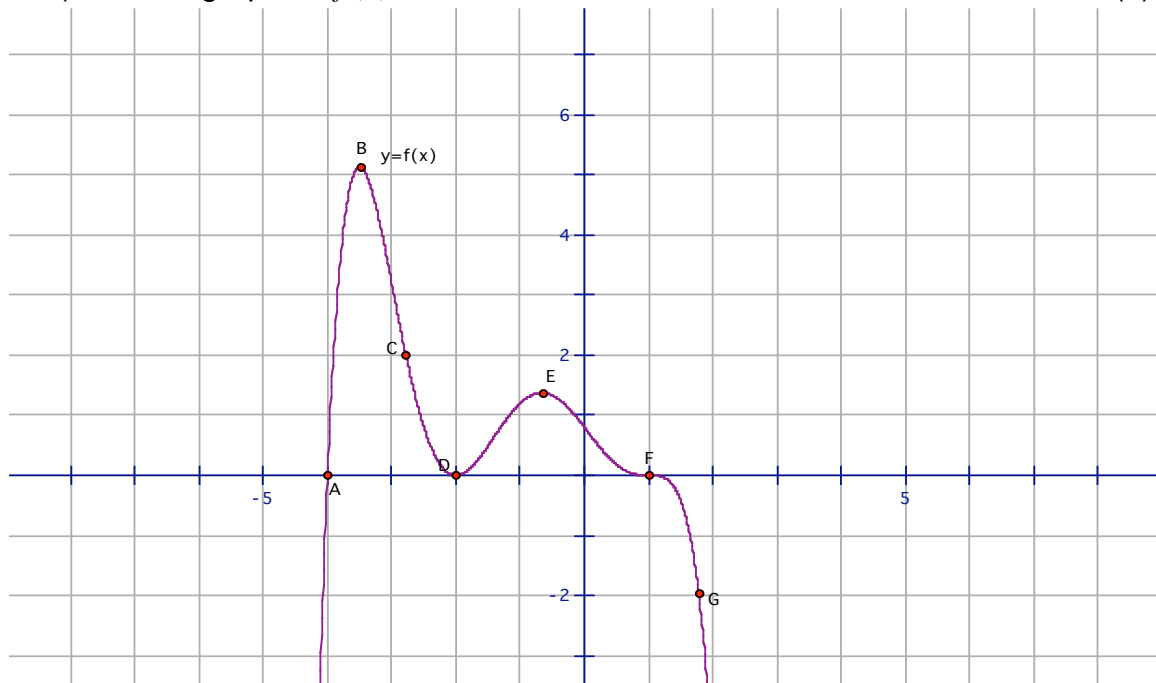
5. Shown below is the graph of the **second derivative** of a function,  $f''(x)$ .  
 At  $x=2$ , the graph of  $f(x)$  is : (one mark)

- a) concave down    b) concave up    c) at a local max  
 d) at a local min    e) does not exist



**Section C-**

1. a) For the graph of  $f(x)$  shown, fill in the table below with + or – or 0. (7)



Point	$f(x)$	$f'(x)$	$f''(x)$
A			
B			
C			
D			
E			
F			
G			

b) Use the graph to estimate  $f'(0)$ : (1)

2. a) It is known that the graphs  $f(x) = \frac{ax - 1}{2x + 1}$  and  $g(x) = bx^2 + 5x - 3$  are tangent to one another at  $x = -1$ . Find the values of  $a$  and  $b$ .

b) Find the equation of the common tangent at  $x = -1$

c) Where do the graphs of  $f(x)$  and  $g(x)$  intersect each other again?  
(10 marks total)

3. a) Show that the point  $P(8,-1)$  does NOT lie on the curve  $y = \frac{1}{x}$  (1)

b) Find the equation of the tangent(s) to  $y = \frac{1}{x}$  which pass through point P. ( 5)

4. a) Find the slope of the tangent to  $f(x) = 2\sqrt{5+x}$  **using first principles.**(4)

b) Consider the function  $h(x) = \frac{f(x)}{g(x)}$ . Find a possible function  $g(x)$  such that  $h(x)$  has a horizontal tangent at  $x = -1$ . (3)

Note: As before,  $f(x) = 2\sqrt{5+x}$