Introduction to Algebra

3.1 What is Algebra?

Algebra is generalized arithmetic operations that use letters of the alphabet to represent known or unknown quantities. We can use y to represent a company’s profit or the costs of labour. The letters used to hold the places for unknown quantities are called variables, while known quantities are called constants. Variables denote a number or quantity that may vary in some circumstances.

Algebra now occupies the centre of mathematics, because it could be used to solve a variety of complex problems much faster than using arithmetic methods. Many problems that mathematicians could not solve previously with arithmetic methods can now be solved with algebraic methods. As well, algebra has made it possible to apply mathematics in other areas of human endeavour such as economic planning, pharmacology, medicine, and public health.

Consider this, 12 + b = 20. What is the value of b? The only quantity that can take the place of b is 8, because 12 + 8 = 20. So 8 is the true replacement value for b.

What about y + y = 15? The replacement value for the first y could be any number not more than 15. However, the replacement value we pick for the first y will determine the value for the second y. If we say, for example, that the first y is 10, then the second y must be 5. As well, if the first y is 12, the second y must be 3. The reverse is also true. Try it for yourself by picking a replacement value for the second, and determine the value for first y. As we will see later, this simple example is very important for understanding the solution to equations involving two similar variables.

Consider another example, 4x + 8 = 40. In this example, we are looking for a number when multiplied by 4 and added 8 to it will give 40. We can try to figure out this number through guessing and checking. Eventually we will find that 8 is the replacement for x. However, with a systematic procedure of solving equations, we can easily solve that problem without going through the throes of guessing and
Before we start learning that procedure for solving equations, let us try to understand the meanings of like terms and unlike terms.

### 3.2 Like Terms and Unlike Terms

Consider again, \(12 + b = 20\). A number or letter separated by the operation sign + is called a term. So 12 is a term; \(b\) too is a term. Letters of the same kind in an algebraic statement or expression are called like terms. For example, \(b + b + c = 2b + c\).

The two \(b\)s are **like** terms, so we can carry out the operations of addition on them.

Look at more examples below.

1. **eg2** \(2b + 3b = 5b\), whatever the value of \(b\) is.
2. **eg3** \(3y + 5x\) cannot be simplified two terms are different.
3. **eg4** \(10t - 4t + 12y + 6t = 6t + 6t + 12y\) (since \(10t - 4t = 6t\))
   
   \[= 12t + 12y\] (since \(6t + 6t = 12t\))

   This is now fully simplified, since the two remaining terms are not like terms.

4. **eg 5** \(3t - 2t = t\). Note: traditionally, mathematicians do not write \(1t\). They just write \(t\). So, \(t\) understandably means \(1t\) (one \(t\)).

The general rule, however, for adding and subtracting like terms does **not** apply to multiplication or division. In fact, multiplication and division are done as is shown in the following examples.

**Multiplication/Division examples:**

1. **eg 1** \(y \times y \times y = y^3\)
2. **eg2** \(x^2 \times x^3 = x \times x \times x \times x \times x = x^5\)
3. **eg 3** \(t \times t = t^2\)
4. **eg 4** \(4t \times 6t = (4 \times 6) (t \times t) = 24t^2\).

**Note:**

1. \(26t\) is the same as \(26 \times t\).
2. \(30yz = 30 \times y \times z\)
3. \(48x^2 = 48 \times x \times x\)
4. \( 2t \div t = \frac{2t}{t} = 2 \), because the two \( t \) cancel themselves out.

5. \( 14x \div 7 = \frac{14x}{7} = 2x \), because 7 goes into 14 two times.

6. \( 20xy \), is the same as \( 20 \times x \times y \). Again, there is no need to put the multiplication sign between 20 and \( x \) or \( x \) and \( y \).

7. \( y \times y \times y \times x \times x = y^3x^2 \)

8. \( 5n^3t^3 = 5 \times n \times n \times t \times t \times t \)

9. To simplify \( \frac{3a^2 \times 4a^3}{2a} \), first multiply the numbers together to get 12 from \( (3 \times 4) \) and then add the exponents to get \( a^{2+3} = a^5 \). This is the same as \( a \times a \times a \times a \times a = a^5 \). That is, keep the base \( a \) and add the exponents. We now have \( 12a^5 \div 2a \). Divide the numbers, \( 12 \div 2 = 6 \). Then subtract the exponents when dividing, so \( a^5 \div a^1 = a^{5-4} \). This equals \( a^4 \). This is the same thing as \( a \times a \times a \times a \times a \div a = a^4 \). The final answer is \( 6a^4 \)

10. \( \left( \frac{2}{t} \right)^2 = \left( \frac{2}{t} \right) \left( \frac{2}{t} \right) = \frac{4}{t^2} \) note: We multiplied the top numbers to get 4 (\( 2 \times 2 \)) and the number letters to get \( t^2 \).

11. \( \left( \frac{1}{3} \right)^3 = \left( \frac{1}{3} \right) \left( \frac{1}{3} \right) \left( \frac{1}{3} \right) = \frac{1}{27} \) note that the exponent 3 tells us the number of times the fraction should be multiplied. We multiplied the top numbers to get 1 (\( 1 \times 1 \times 1 \)) and then the bottom numbers to get 27 (\( 3 \times 3 \times 3 \)).

13. \( 5^3 = 5 \times 5 \times 5 = 125 \)

3.3 Substitution and Formula

When we put quantities in place of variables it is called substitution.

Consider this, what is \( b + c + d \), where \( b = 4 \), \( c = 2 \), \( d = 3 \)?

We simply substitute each variable with its corresponding quantity.

\[ b + c + d = 4 + 2 + 3 = 9 \]
Example 1
Evaluate, 6x + 8y + 3z, where x = 2, y = 3, z = 4
= 6(2) + 8(3) + 3(4)
= 12 + 24 + 12 = 48

Example 2
Evaluate 12a ÷ 14b, where a = 3, b = 1
= 12(3) ÷ 14(1)
= 36 ÷ 14 = \frac{36}{14} = \frac{18}{7} \text{ (dividing by 2, which is the common factor)}

Example 3
Evaluate 2(x - 3) = 10, if x = 8
Remember the distributive property! 2(x - 3) = 10
2(x) - 2(3) = 10 (Distribute 2 over x and -3 in order to remove the bracket)
2x - 6 = 10
2(8) - 6 = 10 (substituting 8 for x)
16 - 6 = 10

A formula describes how one quantity relates to one or more other quantities. A formula is a shorthand form of procedure for doing calculations. Al-Khwarazmi gave the world algebra by using letters of the alphabet to represent unknown numbers. And, then, he gave these symbols all the properties of numbers. Before this, the only way to know a procedure is to see examples of the procedure for specific numbers. For example, procedure for finding the area of a rectangle was taught by showing the procedure for calculating the area for specific rectangles. In a sense, arithmetic reasoning was more of experience rather than deductive reasoning.

However, once a formula is constructed, it can be used to calculate unknown quantities. Formulas too are used to calculate quantities in businesses. A common formula is \( P = R - E \), where \( P \)= profit, \( R \)= revenue (sales), and \( E \)= Expenses.
The formula says that profit is equal to revenue subtract expenses. Another common formula is $E = A - L$, where $E$ = owner’s equity (the capital the owner has invested in the business), $A$ = assets (Resources such as machines, furniture, buildings, etc.), and $L$ = liabilities are amounts owed to others. As well, when $S = VC + FC$, this is called the break-even point, where $S$ = sales, $VC$= variable cost, and $FC$ = fixed cost. The following are other formulas used in businesses:

- Net sales = Gross sales – sales returns and allowances
  \[ NS = GS - SRA \]

- Cost of goods sold = Cost of beginning inventory + cost of purchases – cost of ending inventory

- $VC = \text{Cost per unit} \times \text{Quantities produced}$
  \[ VC = CPU \times Q \]

- Total cost = $VC + FC$, or $TC = VQ + FC$

The Construction of a formula is a straight-forward process. First, you have to know the purpose for which you want to use the formula. Second, you have to understand the information you will be using to construct the formula. Third, with little knowledge of algebra, particularly substitution, you should have no problem constructing a formula. Lastly, test your formula to make sure that it works.

**Example 1**

A technician charges a basic fee of $40.00 for a house visit plus $15 per hour, when repairing central heating system. Construct a formula for calculating three hours of the technician’s charge.

**Solution**

Let $C =$ charge, and $n =$ number of hours worked. The charge is made up of a fixed cost of $40 and $15 times the number of hours. This translates into the following:

\[ \text{Charge} = 30 + 15n \]
Note: no need to write the multiplication sign between 15 and n.
To calculate the charge for three hours we substitute 3 in place of the n.

\[ C = 30 + 15 \times 3 = 30 + 45 = \$75 \]

Therefore, the charge for three hours is $75.00.

**Example 2**

A window cleaner charges a fee of $20 for visiting a house and $10 for every window he cleans.

a) Write a formula for finding the total cost, C, in dollars, when \( n \) windows are cleaned.

b) Find \( C \) if \( n = 8 \)

c) If the window cleaner wants to earn an income of $220 a day, how many windows does he have to clean?

**Solution**

a) Total cost = fixed charge of $20 plus $10 times the number of windows cleaned.

\[ C = 20 + 10n \]

b) \[ C = 20 + 10 \times 8 \] by substituting \( n=8 \)

\[ C = 20 + 80 = \$100 \]

c) \[ C = 20 + 10n \] is the original equation we constructed.

\[ 220 = 20 + 10n \] (divide each term by 10)

\[ 22 = 2 + n \]

\[ 20 = n \]

Therefore, the window cleaner must clean 20 windows in order to earn $220 a day. However, this question assumes that the window cleaner will get enough cleaning job to earn the $220.00.

**Example 3.**

Julie earns \( x \) in her first year of work with AP Films Ltd. Her salary is increased by $650 every year. How much will she earn in a) the 4th year? b) the 6th year? c) the nth year?
Solution

a) 1\textsuperscript{st} Year Salary = $x + $650(n-1) (x is her earning, and n is number of years)
   Note that she does not get any raise in the first year that is why we have n – 1.
   4\textsuperscript{th} Year Salary = $x + 650(4-1) \quad \text{(Remember: she gets $650 increase every Year, excluding the first year.)}
   = $x + 650 (3)
   = $1,950.00

b) 6\textsuperscript{th} Year Salary = x + 650 (6-1)
   = x + 650(5)
   = x + 3,250

c) 10\textsuperscript{th} Year Salary = x + 650 (10 - 1)
   = x + 650 (9)
   = x + 5,850

Example 4
The cost of framing a photograph is calculated using the formula,
\[ C = 0.55 (l + b) \]
Where \( l \) = length in centimetres
\( b \) = width in centimetres
\( C \) = cost in dollars

a) Find the cost of framing a photograph whose dimensions are 12cm by 16 cm.

b) A photograph with length twice the width was framed for $33.00. How much were the length and the width of the photograph?

a) The formula is \( C = 0.55 (l + b) \)
   \[ = 0.55 (16 + 12) \]
   \[ = 0.55(28) = $15.40 \]
   It costs $15.40 to frame the photograph.

b) Let \( y \) represent the width and \( 2y \) represent the length, since the length is 2 times the width. The formula is \( C = 0.55 (l + b) \)
By substitution, we get

\[ 33 = 0.55 (y + 2y) \]

\[ 33 = 0.55 (3y) \quad \text{(since } 2y \text{ and } y \text{ add up to } 3y) \]

\[ 33 = 1.65y \]

\[ \frac{33}{1.65} = y \]

\[ 20 = y \]

Therefore, the width is 20 cm. The length is 40 cm or 20 x 2.

Note that once we have a formula we can use it for evaluating any values we want.

However, you should always remember that constructing a formula involves three steps. First, read the information you are going to use to construct the formula carefully. Second, use letters of the alphabet in constructing the formula, bearing in mind its purpose. Finally, test the formula to make sure that it works for the purpose for which you constructed it.

### 3.4 Equations

An equation is a mathematical statement or expression in which two quantities are equal. Similar to formulas, equations also use numbers, letters of the alphabets and operational symbols (+, -, \(\times\), ÷). In fact, formulas are special form of equations.

**Example 1**

Do you know that \(2 + 5 = 7\) is an equation? The equality sign (=) separates the statement into left side and right side. The sum of the left side is \(2 + 5 = 7\). This is equal to the number on the right side.

**Example 2**

Solve for N if \(N + 15 = 25\).

This is an example of an equation, because it is separated by the equality sign (=) and there is a left side and a right side. The equation means a certain number \(N\), plus 15 equals 25. The replacement value for \(N\) must be such that when added to 15, it will be equal to 25. By inspection or guessing and checking, you will know that \(N\)
must be 10. So, 10 + 15 = 25. However, in some cases, the replacement value for a variable may not be as simple as in this example. Accordingly, mathematicians have developed a more sophisticated procedure for solving an equation.

Solve for N: \( N + 15 = 25 \). When we subtract 15 from both sides of the equation,

\[
\]

Note that \(+15 - 15 = 0\), because they are inverse of each other. This procedure helps to isolate the \( N \) on the left side of the equation. The right side becomes 10.

So we are left with, \( N = 10 \) as we figured out before. This procedure leads to the following rule:

*When the same number is added to or subtracted from both sides of an equation, the equation does not change.*

**Example 3**

Solve for \( x \), if \( 2x + 3 = 15 \).

The above equation reads \( 2 \) times a certain number plus 3 equals (or the same as, or equivalent to) 15. The \( x \) as a variable is a placeholder. Also, remember that there is no need to write the multiplication sign (\( \times \)) between 2 and \( x \).

To solve \( 2x + 3 = 15 \), our aim is to isolate the variable on the left side of the equation. So we subtract 3 from both sides of the equation.

\[
2x + 3 - 3 = 15 - 3
\]

\[
2x + 0 = 12
\]

We now have, \( 2x = 12 \). Again, this means \( 2 \) times a certain number equals 12.

Certainly, we can guess and check to find that \( 6 \) is the replacement value for \( x \).

However, if we divide both sides of the equation by 2 we have the following situation.

\[
\frac{2x}{2} = \frac{12}{2}
\]

So, \( x = \frac{12}{2} = 6 \).
This leads us to the next rule of equation. \textit{It says that when both sides of an equation is divided or multiplied by the same number the equation does not change.}

Note that we can check to make sure that 6 is truly the replacement value for \(x\).

\[2x + 3 = 15\] which is the original equation

\[2(6) + 3 = 15\] substitute \(x\) with 6 (Remember substitution?)

\[12 + 3 = 15\] Multiply 2 and 6

\[15 = 15\] Add 12 and 3 to get 15. Now you see that 15 equals 15. That means we are right that the replacement value for \(x\) is 6.

\textbf{Example 4}

Solve for \(x\), \(4x - 3 + 2x = 8x - 3 - x\)

\[6x - 3 = 7x - 3\] We simplified the left side by adding 4x and 2x together to get 6x. We also simplified the right side by adding (8x) and (-x) to get 7x.

The basic rule of integer is that when we have a large number that is positive, we always take small negative number from it.

\[6x - 3 + 3 = 7x - 3 + 3\] We added +3 to both sides. Do you remember the first rule of equation? An equation is like a scale. What you do to one side, you do the same to the other side.

\[6x = 7x\] This is the result after we added +3 to both sides

\[6x - 7x = 7x - 7x\] We subtract -7x from both sides to isolate \(x\).

\[-x = 0\] After subtracting -7x from both sides.

\[-x \times -1 = 0 \times -1\] We multiplied each side by -1, so that we get \(x\).

\[x = 0\] Now we are done; \(x\) is equal to 0.

Let us check that the replacement value for \(x\) is 0.
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4x - 3 + 2x = 8x -3 -x was the original equation.

4(0) – 3 + 2(0) =8 (0) – 3 – 0 (We substituted 0 in the place of x)

0 -3 + 0 = 0 -3 -0 is the result after multiplying.

-3 = -3. Since the left side and right side are equal, the replacement value this verifies that x=0 is the correct answer.

3.5 Using Equations to Solve Problems

Equations are a powerful tool used to solve many problems in accounting management, marketing, and the sciences. However, before we use this tool, we have to be familiar with its language. The following table will help you to become familiar with algebraic language.

<table>
<thead>
<tr>
<th>Addition</th>
<th>Subtraction</th>
<th>Multiplication</th>
<th>Division</th>
<th>Equality</th>
</tr>
</thead>
<tbody>
<tr>
<td>The sum of</td>
<td>Less than</td>
<td>Times</td>
<td>divides</td>
<td>Equals</td>
</tr>
<tr>
<td>Plus/total</td>
<td>Decreased by</td>
<td>Multiplied by</td>
<td>Divided by</td>
<td>Is/was/are</td>
</tr>
<tr>
<td>Increased by</td>
<td>Subtracted from</td>
<td>The product of</td>
<td>Divides into</td>
<td>The result</td>
</tr>
<tr>
<td>More than</td>
<td>Difference</td>
<td>Twice/two times</td>
<td>per</td>
<td>What is left</td>
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<tr>
<td>More than</td>
<td>between</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exceeds</td>
<td>Diminished by</td>
<td>Double(two times)</td>
<td>Quotient</td>
<td>What remains</td>
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<tr>
<td>Expands</td>
<td>Take away</td>
<td>Triple (three</td>
<td>Half of</td>
<td>The same as</td>
</tr>
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<td></td>
<td></td>
<td>times)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Greater than</td>
<td>Reduced by</td>
<td>Half times</td>
<td>Quarter of</td>
<td>Gives/giving</td>
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<tr>
<td>Gain/profit</td>
<td>Less/minus</td>
<td></td>
<td>Third of</td>
<td>Makes</td>
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<td>Longer</td>
<td>Loss</td>
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<td>Leaves</td>
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<td>Heavier</td>
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<td>Wider</td>
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<td>Taller</td>
<td>Smaller than</td>
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<tr>
<td>Added to</td>
<td>slower</td>
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</table>
Apart from learning to understand algebra language in terms of addition, subtraction, division, and multiplication, students also have to learn how to change English sentences to algebraic expressions. Since this is not entirely an algebra course, we will provide only a few simple examples of how to change English phrases into algebraic expressions. Essentially, the skills you learn in this section are what you need for both an elementary and an intermediate algebra course. Study the examples below carefully, so that you can spot these English phrases when used in framing problems. As soon as you spot them, then you have to think about how to convert them into algebraic expressions.

<table>
<thead>
<tr>
<th>English Phrases</th>
<th>Algebraic Expressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eight more than five times a number</td>
<td>$8 + 5n$, where $n$ is the unknown.</td>
</tr>
<tr>
<td>Ten times the sum of a number and 5</td>
<td>$10 (y + 5)$, where $y$ is the unknown</td>
</tr>
<tr>
<td>A number subtracted from 50</td>
<td>$50 - x$, where $x$ is the unknown number.</td>
</tr>
<tr>
<td>Four less than a number</td>
<td>$n - 4$, where $n$ is the unknown.</td>
</tr>
<tr>
<td>One-half of a number</td>
<td>$\frac{1}{2}n$, where $n$ is the unknown number.</td>
</tr>
<tr>
<td>Five more than twice a number</td>
<td>$5 + 2x$</td>
</tr>
<tr>
<td>Six less than two-thirds of a number</td>
<td>$\frac{2}{3}n - 6$</td>
</tr>
<tr>
<td>Three-fourths of the sum of a number and 12</td>
<td>$\frac{3}{4}(y + 12)$</td>
</tr>
<tr>
<td>Fifty subtracted from three times a number</td>
<td>$3z - 50$</td>
</tr>
</tbody>
</table>

Now is the time to use equations to solve problems. The first thing we need to do is to read the given information carefully. Second, we have to use a variable to represent the unknowns and then represent one unknown in terms of another. After that, we set up an equation and solve the equation to find the unknown(s). Nine worked examples are provided below to illustrate the power of equations in solving problems.
Example 1

Gabby’s gross monthly income is 2.5 times that of Maggie income. The sum of their gross monthly income is $7,000. What is each person’s gross monthly income?

Solution

Let $y$ be Maggie’s income and $2.5y$ be Gabby’s income. Together their income equals $7,000. So, $y + 2.5y = 7000$. Adding the like terms on the left side

$$3.5y = 7000$$

Dividing both sides by 3.5,

$$y = \frac{7000}{3.5} = 2000$$

So Maggie’s income is $2,000. and Gabby’s income is $5,000. (2.5 x 2,000).

Check the accuracy of the answer by substituting the values in the original equation in the following way:

$y + 2.5y = 700$ is the original equation

$2,000 + 2.5(2000) = 7000$ Since the value of $y$ is 2000, we substitute it in the place of $y$. Remember that $y$ is a variable or a placeholder for 2000.

$2,000 + 5,000 = 7,000$ We multiplied 2.5 by 2,000 to get 5,000.

$7,000 = 7,000$ We added 2,000 and 5,000 to get 7,000. The left side and the right side is the same amount, indicating that our solution is right.

In fact, if we calculate 2.5 of $7000$ we get $5,000$. Again, it shows that our answer is right. So whatever way we check our answer, it indicates that we are right. You must develop this skill of checking the accuracy of your answer after solving an equation.

Example 2

A commuter train has four double decker cars and five regular ones. Each double decker has 68 seats more than a regular one. The total number of seats on the train is 1118. a) What is the number of seats on each type of car? b) What is the total seats in each type of cars?
Solution

- Let \( x \) be the number of seats on a regular car. Then let \( x + 68 \) be the number of seats on a double decker. (Note: This is logical since there are 68 seats more on a double decker than a regular car).
- Since there 4 double decker cars, there must be a total of \( 4 \times (x + 68) \) seats.
- Similarly, since there are 5 regular cars, there must be a total of \( 5x \) seats.
- The given information says that there are a total of 1118 seats on the train.
- The following relationship is given:

  a) \((\text{# of double decker cars}) \times (\text{# of seats on a double decker car}) + (\text{# of regular cars}) \times (\text{# of seats on a regular car}) = \text{the total # of seats on the commuter train.}\)

  In terms of algebraic expression, we have the following:

  \[
  4(x + 68) + 5x = 1118 \\
  4x + 272 + 5x = 1118 \\
  4x + 5x + 272 = 1118 \\
  9x + 272 = 1118 - 272 \\
  9x = 846 \\
  x = \frac{846}{9} = 94
  \]

  Therefore the number of seats in a regular car is 94.
  The number of cars in a double decker car is 162 \((94 + 68)\)

  b) The total seats in the regular cars = 5 \((94)\), since there are 5 regular cars, each with 94 seats. The total number of seats on the four double decker trains = \(4(94 + 68)\) = 648

  Total seats on the commuter train: 648 + 470 = 1118.

  The difference is 68 \((162 - 94)\), indicating that our answer is right.

  Do you have any other ways to check the answer?

  Please, note that we could also check the accuracy of our answer by writing 94 in the place of the \( x \) and solving the equation. Try this yourself.

Example 3

Mr. Martins owns a specialty store. He purchased an equal number of two types of designer phones for a total of $7,200. The top quality phone costs $120 each and the plastic phones costs $80 each.
a) How many of each type of phones were purchased?

b) What was the total value of each type of phone?

Solution

a) Let x be the quantity of top quality phones purchased. x can also represent the number of plastic phones purchased. This is the case because the same quantity of each type of phones was purchased.

The following relationship is true:

\[(\text{# of top-quality phones}) \times \text{(the price)} + (\text{# of plastic phones}) \times \text{(the price)} = \$7200\]

\[
x \times 120 + x \times 80 = 7200
\]

\[
120x + 80x = 7200
\]

\[
200x = 7200
\]

\[
x = \frac{7200}{200} = 36
\]

So, 36 quantities of each phone were purchased.

We can check the accuracy of our answer by substituting 36 in the place of x and evaluating as we did in substitution. Try that yourself as a practice.

b) The dollar value of the Top-Quality phone = Quantity purchased x unit price

\[= 36 \times 120 = \$4,320.00\]

The dollar value of plastic phones = Quantities purchased x unit price

\[= 36 \times 80 = \$2,880\]

Thus, $4,320 + $2,880 = $7,200. This proves the accuracy of our calculations.

Example 4

Focus Day Care Centre has 28 children and 7 care-givers. Two of the care-givers work part-time. Children enrolled under 4 years old are 8 more than those over 4 years old. How many children under 4 years old are enrolled in the centre?
Solution

Let \( n \) represent the number of children over 4 years old.
Let \( n + 8 \) represent the number of children under 4 years old. This so because the number of children under 4 years old are 8 more than those over 4 years old.

This relationship is true:

\[
\text{(\# of children over 4 years old) + (\# of children under 4 years old) = 28}
\]

\[
\downarrow \quad \downarrow
\]

\[
n \quad + \quad n + 8 = 28
\]

\[
2n + 8 = 28
\]

\[
2n + 8 - 8 = 28 - 8
\]

\[
\frac{2n}{2} = \frac{20}{2}
\]

\[
n = 10
\]

The number of children over 4 years old is 10.

Thus, the number of children under 4 years old = \( n + 8 \)

\[
= 10 + 8
\]

\[
= 18
\]

Note that we could interpret the above problem in this way. Since there are 8 more children under 4 years old than over 4 years old, we can say that there are 8 less children over 4 years old than under 4 years old. So let \( n-8 \) be the number of children over 4 years old, then \( n \) can represent those under 4 years old. This will give us the following equation:

\[
n + n - 8 = 28
\]

Try to solve the above equation for \( n \) and compare your answer to the above solution. Did you get the same result? If not, you should check over your work.
Example 5

Industrialized nations have 2,017 radios per thousand people. This is about six times the number of radios per thousand in developing nations. What is the number of radios per thousand in developing countries? (Round off to the nearest whole number).

**Solution**

Let \( r \) represent the number of radios per thousand in developing countries. The following relationship is given in the question:

\[
\text{# of radios in developing countries per thousand x 6 = 2017}
\]

\[
6r = 2017
\]

\[
\frac{6r}{6} = \frac{2017}{6}
\]

\[
r = 336.17
\]

\[
r = 336 \text{ (to the nearest whole)}
\]

So there are 336 radios per thousand in developing countries. We can check the accuracy of our answer in this way: \(6r = 2017\), then \(6 \times 336.17 = 2017\).

Though this question could be solved using other methods, we used an equation to solve it to show that an equation can be used to solve any question. Its power is limitless!

Example 6

Forty acres of land were sold for $810,000. Some were sold for $24,000 per acre and the rest for $18,000 per acre. How much was sold at each price?

**Solution**

The unknowns are the number of acres sold at $24,000 per acre and the number of acres sold at $18,000 per acre.

Let \( L \) represent the number of acres sold at $24,000, then 40 - \( L \) represent the rest sold at $18,000 per acre.
Note the following relationship is implied in the given information.

\[(\# \text{ of acres sold}) \times (24,000 \text{ per acre}) + (\# \text{ of acres sold}) \times (18,000 \text{ per acre}) = 810,000\]

\[(L) \times (24,000) + (40 - L) \times (18,000) = 810,000\]

\[24000L + 720000 - 18000L = 810000\]

\[6000L + 720000 = 810000\]

\[6000L = 90000\]

\[L = 15\]

Therefore, 15 acres of the land were sold at $24,000. And 25 acres (40 - 15) were sold at $18,000 per acre. Check the accuracy of the answer by substituting the two values the original equation.

**Example 7**

A bank pays the following interest on an account.

<table>
<thead>
<tr>
<th>Month</th>
<th>interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$30</td>
</tr>
<tr>
<td>2</td>
<td>$50</td>
</tr>
<tr>
<td>3</td>
<td>$70</td>
</tr>
<tr>
<td>4</td>
<td>$90</td>
</tr>
</tbody>
</table>

a) Write a formula or equation relating the month to the amount of interest paid.

b) How much interest will the bank pay on the 10th month?

**Solution**

a) We have to examine the table for any patterns. If we subtract the first month interest of $30 from the second month interest of $50, we get $20. Again, if we subtract the second month interest of $50 from third month interest of $70, we get $20. If seems that this pattern is true in the table- there is a constant difference of
Further, if we multiply the constant difference of $20 by the month and add 10 we get the amount of interest. Let us try this and see if it works.

Month x 20 + 10 = interest

1 x 20 + 10 = 30  first month interest
2 x 20 + 10 = 50  second month interest

Therefore, let m represent the month and i the interest. We can write a formula based on the above relationship.

\[ 20m + 10 = i \]

b) For the 10th week, the bank will pay,

\[ 20m + 10 = i \]

\[ 20 (10) + 10 \]

\[ = 200 + 10 \]

\[ = 210 \]

The bank will pay $210.00

**Example 8**

Modern Restaurant sells pizza, meat pies, and chicken wings. The chart below shows the price of chicken wings.

<table>
<thead>
<tr>
<th>Wings</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 wings</td>
<td>$1.99</td>
</tr>
<tr>
<td>10 wings</td>
<td>$3.99</td>
</tr>
<tr>
<td>15 wings</td>
<td>$5.99</td>
</tr>
</tbody>
</table>

a) Based on the information in the above table write a formula to show the relationship between the price and the number of chicken wings.

b) How much will 50 chicken wings cost?
Solution

a) Let’s examine the pattern in the table. Though the difference between $3.99 and $1.99 is $2, the pattern does not look like example 7. This is because if we multiply the number of wings by 2 we get 10, the price is far greater than $1.99. This is mainly because the numbers in the left column are not consecutive integers. So, let’s divide 1.99 by 5. We get 0.40. Let’s try if this works. 0.40 x 5 = 2.00. We need to subtract 0.01 from $2 to get $1.99. Let w represent the number of wings, and C the total cost. Therefore, the formula becomes, $0.40w – 0.01 = C. Let’s test this formula.

How much will 15 wings cost?

$0.40w – 0.01

0.40(15) – 0.01 = C

6 – 0.01 = $5.99, this is the correct price in the chart.

b) 50 wings cost,

$0.40w – 0.01 = C

0.40 (50) – 0.01 = C (substitute 50 for w)

20 – 0.01 = $19.99

50 wings cost $19.99

Example 9

A company’s annual profit of $8,400,000 was allocated to reserve fund, dividend distribution, and operation expansion. The amount allocated to dividend distribution was twice the amount allocated to the reserve fund. The amount allocated to operation expansion was $400,000 more than the amount allocated to dividend distribution. Find the amount of each allocation.
Solution

The first step is to identify the unknowns. These are the amounts allocated to reserve fund, dividend distribution, and operation expansion. However, we are told that the amount for dividend distribution was twice that of reserve fund, and operation expansion allocation is 400,000 more than dividend amount. Therefore, we can let $A$ represent the amount for the reserve fund, $2A$ can represent dividend amount, and $2A + 400,000$ for operation amount. The following relationship is given:

$$(\text{Reserve fund}) + (\text{dividend distribution}) + (\text{operation expansion}) = 8,400,000$$

$$A + 2A + 2A + 400,000 = 8,400,000$$

$$5A + 400,000 = 8,400,000$$

$$5A + 400,000 - 400,000 = 8,400,000 - 400,000$$

$$5A = 8,000,000$$

$$\frac{5A}{5} = \frac{8,000,000}{5}$$

$$A = 1,600,000$$

Thus, $1,600,000$ was allocated to reserve fund.

Dividend distribution: $2A = 2 (1,600,000) = $3,200,000$

Operation expansion: $2A + 400,000 = 2 (1,600,000) + 400,000 = $3,200,000 + $400,000 = $3,600,000$.

Check the allocation: $1,600,000 + $3,200,000 + $3,600,000 = $8,400,000$. 
3.6 The Concept of Break-Even Point

Break-even point occurs when revenue equals expenses or cost. At that point, a business organization does not make any profit or loss. That is, sales = Fixed expenses + variable expenses. In this case, sales are just enough to cover expenses without incurring a loss. The data below illustrates the concept of break-even.

<table>
<thead>
<tr>
<th></th>
<th>May</th>
<th>June</th>
<th>July</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>$25,000</td>
<td>$85,000</td>
<td>$125,000</td>
</tr>
<tr>
<td>Less: Variable cost</td>
<td>10,000</td>
<td>75,000</td>
<td>115,000</td>
</tr>
<tr>
<td>Contribution</td>
<td>15,000</td>
<td>10,000</td>
<td>10,000</td>
</tr>
<tr>
<td>Less: fixed costs</td>
<td>10,000</td>
<td>10,000</td>
<td>10,000</td>
</tr>
<tr>
<td>Net profit</td>
<td>$5,000</td>
<td>$0</td>
<td>$0</td>
</tr>
</tbody>
</table>

From the above table, you will see that the company broke even the month of June and July- it did not make any profit or loss; Sales were just enough to cover variable costs and fixed costs.

The calculation of the break-even point is very important for at least two reasons:

a) It helps business owners or entrepreneurs to plan for profit or analyze loss.

b) It helps business owners to set prices at which they will sell their products.

c) It can also be used to predict profit or loss in any business ventures.

To calculate the break-even point, we have to be familiar with certain terms, which are defined below:

**Variable costs** or expenses are the costs of labour, utilities, raw materials and any other expenses directly traced to production of goods or services. An important characteristic of variable cost is that it increases as production increases and decreases as production decreases. However, the nature of a business organization will determine what will constitute its variable cost. For example, produce grocery business’s variable cost will consist of the cost produce, shipping, storage, spoilage, and wages of cashiers.
**Fixed costs** or **expenses** are those that remain unchanged at least in the short or medium term. They are not directly traceable to the production of goods or services. Rather, fixed cost is termed a period cost and is spread over the units produced or the volume of services rendered. In the case of produce grocer, the fixed cost may consist of rent of premises, insurance, taxes, heating and cooling costs, salaries of supervisors, administrative expenses, and cleaning.

**Unit contribution margin** is the amount left after deducting variable cost per unit from the revenue per unit. Note that some times, we may not have any contribution margin and we have to use figures such as the number of people or the cost of an item. This is the case with many performance art shows, where the owners have to do some calculations to find out if it is profitable to hire a performer. The two examples below show how to calculate a break-even point. The first uses contribution margin, while the second does not.

**Example 1**

M A Berry Farm produces black berry at a variable cost of $2.40 per basket and sells it to a local farmer’s market for $4.00 a basket. M A Berry has $2,000 fixed cost for the berry operation.

a) How many baskets does it need to produce in order to break even?

b) How many baskets should the farm produce if it desires profit of $3,000?

**Solution**

a) Unit contribution margin (UC M) = $4  $2.40 = $1.60

\[
\text{Break-even in units} = \frac{\text{Fixed cost}}{\text{UCM}} = \frac{2000}{1.60} = 1,250 \text{ baskets.}
\]

Thus, the Break-even units are 1,250 baskets. When M A Berry produces these baskets and is able to sell them at $4.00 a basket, it will not make any profit or incur loss. Note
that at the Break-even baskets, \( S = VC + FC \), where \( S = \) Sales, \( VC = \) Variable cost, \( FC = \) Fixed Cost. \( S = 4 \times 1,250 = 5,000 \), \( VC = 2.40 \times 1250 = 3000 \), \( FC = 2,000 \).

\[ S = 5,000 = 3,000 + 2,000. \]

Alternatively, we can calculate the break even point algebraically. Let \( y \) be the number of baskets of berries that must be sold to break even.

Sales are \( 4 \times y = 4y \); Variable cost \( = 2.40 \times y = 2.40y \); fixed cost \( \$2000 \)

The equation is \( S = VC + FC \). Substitute the values in place of the letters.

\[
\begin{align*}
4y &= 2.40y + 2000 \\
4y - 2.40y &= 2.40y - 2.40 + 2000 \\
1.60y &= 2000 \\
y &= \frac{2000}{1.60} = 1,250 \text{ baskets}
\end{align*}
\]

b) At a planned profit of \( \$3,000 \), the Break-even point \( = \frac{FC + P}{UCM} \), where \( P = \) profit. We have to add the profit to the fixed cost because we treat it as something we have to recover.

\[
\text{Break-even point} = \frac{2000 + 3000}{1.60} = \frac{5000}{1.60} = 3,125 \text{ baskets}
\]

The farm needs to produce 3,125 baskets in order to earn a profit of \( \$3,000 \) and cover fixed cost of \( \$2,000 \). That is, it has to produce 1875 baskets more than the previous break-even baskets. Try to rework this problem using algebra.

**Example 2**

At the Christmas time, the final marketing majors consisting of 20 students decided to have a party. The organizers estimated that it would cost \( \$720 \) to have the party.

a) How much should a ticket be sold in order to break-even?

b) The organizers decided to beat down the cost per ticket by allowing students to invite their family members, friends, and significant others to the party. It was anticipated that this will bring in additional 10 people. With this information, how much should a ticket be sold in order to break-even?

c) The organizers decided that since they are responsible for any furniture that might be broken or damaged walls, they have to charge something to make provision for
that eventuality. As a result, they agreed to add $200 to the cost. With this additional information, how much should a ticket be sold to break-even?

d) The student union has agreed to contribute $300.00 toward the party. With this additional information, how much should a ticket cost?

Solution

a) Break-even point = \( \frac{\text{Fixed Cost}}{\text{Contribution}} = \frac{720}{20} = $36.00 \). A ticket should be sold for $36 in order to cover cost. The student may be surprised by the formula used above. The main idea behind break-even concept is how to cover cost without incurring loss or profit. Since the given information does not include variable cost, the denominator of the break-even formula can simply be written as contribution. This immediately suggests that 20 people are going to contribute equally to pay for the $720.00.

b) Break-even point with additional 10 people = \( \frac{720}{30} = $24 \). Thus, a ticket should sell for $24.00 to cover cost.

c) In this case, we add the $200 to the cost of the party as part of the expense to cover. The break-even point = \( \frac{720 + 200}{30} = \frac{920}{30} = $30.67 \)

d) If the student union agreed to contribute $300 toward the party, the cost will be reduced by that amount. The new breakeven point = \( \frac{920 - 300}{30} = \frac{620}{30} = $20.67 \). The new contribution has reduced the break-event ticket price to $20.67.

Use algebra to solve parts a and b of example 2 to see if you will get the same results. If you did not get the same results, carefully check your work. If that does not help, then go over the section on equations to review.
3.7 Reviews, Exercises, and Assignments

1. Tickets for a school dance are sold at $12 for students and $9 for professors.
   a) If p students and q professors buy tickets, write a formula for the total value, $T$, of the ticket sales.
   b) Find the total value of the ticket sales if p = 50 and q = 20.

2. The profit made by a salesperson when he makes sales on a day is calculated with the formula, $P = 4n - 5$, where n = sales, P= Profit.
   Find the profit if he makes a) 30 sales b) 20 sales c) 9 sales.

3. Fred sells bicycles on a salary plus commission basis. He receives a salary of $500 every two weeks and a commission of $25.00 for each bicycle that he sells. How many bicycles must he sell in a month in order to have a monthly income of $1750?

4. Pat is paid time-and-half for each hour worked over 36 hours in a week. Last week she worked 42 hours and earned $495.90. What is her normal hourly rate of pay?

5. A plumber repair bill, not including taxes, was $195. This included $34 for parts and an amount of 3 hours of labour. Find the hourly rate that was charged for labour.

6. Doug is paid double time for each hour worked over 40 hours a week. Last week, he worked 46 hours and earned $668. What is his normal hourly rate?

7. Jay earns $p in his first year of employment. The following year his salary is increased by $x. Write a formula for his salary in his second year.

8. Gasoline costs $x cents per litre. Write a formula for the cost, C cents of $g$ litres of gasoline.

9. Sharon works for 40 hours a week. Her rate of pay is $12 per hour plus a vacation pay of $5 per hour.
   a) Write a formula to calculate her pay. Let h=#hours worked.
b) Calculate total pay for a regular week of 40 hours work.
c) If Sharon is paid time-and-half for overtime, rewrite the formula in a) to include overtime. Let H = # of overtime hours.

10. The formula \( P = 350n - 200 \), gives the profit \( p \), made when \( n \) cars are sold in a day at a show room. The $200 is the rental cost of the show room per day.
   Find \( P \) if a) \( n = 2 \), b) \( n = 3 \) c) \( n = 4 \) d) \( n = 10 \).

11. A plumber charges $50 for a house visit and $40/hour for any repairs made, excluding the cost of materials. The formula is, \( B = 50 + 4h \), where \( h \) is the hours worked and \( B \) is the total bill. Find the total bill if the plumber works,
a) 0 hours b) 1 hour c) 3 hours d) 5 hours e) 5.5 hours.

12. Bracing Parking Lot Inc. has the following sign displayed at the entrance of its parking lot:

<table>
<thead>
<tr>
<th>Parking Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>½ hour or less</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>

   Maximum
   8:00 am to 9:00 am $15.00
   (Monday to Friday)

   a) Write a formula for calculating the total parking fee. Let \( P \) stand for the total parking fee, and \( h \) the hours parked.
b) Use the formula you constructed in (a) to calculate how much Edna paid for parking for three hours.
c) Fidelia paid $54.00 for parking. How long did she park?
d) John parked for 45 min. How much did he pay?
13. A neighbour charges $110.00 for each cord of firewood plus a $50.00 delivery fee.
   a) What will the neighbour charge for delivering:
      i) 3 cords?    ii) 5 and half cords?    ii) 6 cords?
   b) Write a formula for this problem. Define any variables you will use.
   c) Use your formula to find out how many cords of firewood you will receive if the neighbour charged you $875.
   d) Use your formula to find out how many cords of firewood you would receive if the neighbour charged you $270.

14. A super discount store sold plastic cups for $3.50 each and ceramic cups for $4.00 each.
   a) If 400 cups were sold for a total of $1,458, how many cups of each type were sold?
   b) What was the dollar value of each type of cup sold?

15. Net profit = Gross profit − operating expenses. If net profit is $30,000 and operating expenses are $15,900, find the gross profit.

16. Explain how the formula, Gross profit = net sales − cost of goods sold, can be rearranged to find net sales.

17. At a break-even point of 800 units sold, white company’s variable expenses are $8,000 and its fixed expenses are $4,000. What will the company’s net income be at a volume of 810 units?

18. An appliance dealer sold more washing machines than did dryers. A washing machine sells for $480 and dryer for $350. If the total dollar sales were $21,750, how many of each appliance was sold? What was the total $ amount for the washers? What about the dryers?

19. The manager of Zoom Circus is planning many tickets must be sold to ensure that a matinee performance will make a profit for the circus. The manager knows from
records that the number of children under 12 attending a matinee will be four times the number of adults attending. The total expenses associated with a matinee are $689 per performance. If adult tickets are $5 and children tickets are $2, how many of each must be sold in order to break-even?

20. Evaluate the formula \( S = C + M \), for \( S \), if \( C = $296 \) and \( M = $150 \).

21. Solve for \( x \) if \( 3x - 4 + 2x = 11 \). Check your answer.

22. Eastern University estimates that the cost of running an on-line business course is $8000. This expense is made up of lecturer’s salary, materials, and administrative expenses. If it charges $510 for the course, how many students should enroll in the course in order to break-even?

23. Evaluate the formula \( G = hr \), if \( h = 4 \) hours and \( r = 9.83 \).

24. \( TC = VC + FC \), find \( TC \), total cost, when \( VC = $7,000 \) and \( FC = $4,200 \).

25. Solve and check your answer for \( y \) if \( 3y - 5 = 10 \).

26. Intel Company Ltd. manufactures widgets. Each widget retails at $5. It costs $2.50 to make each and the fixed cost for the period is $750.
   a) How many widgets should Intel make in order to break-even?
   b) What is the break-even point sale revenue?
   c) What is the profit margin if the company produces 400 widgets?

27. Find the indicated variable in each of the cases below.
   a) \( A \): if \( 2A + 5A = 35 \)
   b) \( C \): if \( \frac{C}{2} - 1 = 9 \)
   c) \( K \): if \( \frac{K}{2} + 3 = 5 \).

28. Remove the bracket and solve each equation.
   a) \( 3(B - 1) = 21 \)
   b) \( 2(x - 3) = 6 \)
   c) \( 6(x + 2) = 30 \)
   d) \( 1(y - 48) = 36 \)
29. Yen owns a small retail business that sells T-Shirts. A T-shirt sells for $20 and it costs Yen $10.00. Her fixed or period expense (rent, utilities, and insurance) averages $4000/month.
   a) How many T-shirts should she sell a month in order to break-even?
   b) How many T-shirts should she sell to have a gross sale of $20,000?
   c) What is her net profit if she sells $20,000 worth of T-shirts?

30. Complete the following chart, which represents the monthly operation information about four businesses.
   b) What is Banti Auto’s gross margin per service?
   c) What is A&H Bakery’s total cost for the month?
   d) For Parly Restaurant, how much of its unit price is net profit?
   e) How much of Multi Computer Service’s unit service price is fixed cost?

<table>
<thead>
<tr>
<th>Companies</th>
<th>Total Revenue</th>
<th>Price per unit/service</th>
<th>Units sold/service</th>
<th>Fixed cost</th>
<th>Variable cost per unit</th>
<th>Total Net profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Banti Auto</td>
<td>$120</td>
<td>40</td>
<td>$2,000</td>
<td>$50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parly Rest.</td>
<td>$12</td>
<td>500</td>
<td>$1,500</td>
<td>$5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AH Bakery</td>
<td>$2</td>
<td>3000</td>
<td>$2,200</td>
<td>$0.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ComputerM</td>
<td>$30</td>
<td>240</td>
<td>$1,800</td>
<td>$12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

31. Mat E software manufacturing company has monthly period or fixed cost of $4,913, and variable cost per unit of $60.00. On the average, software sells for $200.
   a) What is the gross profit margin?
   b) What is the break-even point in units?
   c) What is the net profit for selling 1700 units of software?
   d) What is the break-even point in dollars?
32. A house and lot cost $166,000. The cost of the house is 10 times the cost of the lot. What is the cost of the lot?

33. At a soccer game, the admission price is $10 for regular seats and $12 for front seats. The total receipts were $175,000 for 16,300 admissions.
   a) What is the number of regular seat tickets that were sold?
   b) What is the number of front seat tickets that were sold?
   c) What is the revenue realized from the sale of front seat tickets?

34. Joe works as a salesperson at Max Fashion. Last week he sold three times many tie-dyed T-shirts as silk-screened shirts. He sold 176 shirts altogether. How many tie-dyed shirts did he sell?

35. Elaine sold 4 times as many newspaper subscriptions as Rob did. Rob sold 16 fewer subscriptions than Elaine did. How many subscriptions did each sell?

36. Jenetta supervises six times as many data entry clerks as Edna. There are ten fewer clerks working under Edna than under Jenetta.
   a. How many clerks are working under Edna?
   b. How many clerks are working under Jenetta?

37. Solve each equation below.
   a) \( n + 1 = 1 \)  
   b) \( 5a - 3 + 2a = 1 \)  
   c) \( 7x = 49 \)  
   d) \( 3x + 2 = 23 \)  
   e) \( 3x + 5x = 48 \)  
   f) \( x - 56 = -42 \)  
   g) \( 53 + a = 65 \)  
   h) \( 2y - 3 + 5y = y + 15 \)

38. Write the following in algebra. a) Four increased by twice a number.
   b) Ten times the difference of a number and 14.
39. The cost per case of Brown Soft Drink is $1.25. If the total output is 2500 per week, what is the total variable cost?

40. If \( a = 2, b = 3, c = -2 \) and \( d = -3 \), then what is \( \frac{3ab + 4c}{4c - 2d} = ? \)

41. Find the value \( r \) for which \( 3(r-2) - 2(r +1) = 5r + 11 \)

42. During a 5-day community festival, the number of visitors doubled each day. If the festival opened on Thursday with 345 visitors, what was the attendance the Sunday?

43. Production of one unit of star tire requires 20 minutes of a machine tool, whereas production of a unit of jungle tire requires 30 minutes. The machine operated for 47 hours this week and produced 120 units. How many units of each of the two products were manufactured?

44. If \( F = ky \) and \( F \) has the value of 1200 when \( y = 300 \), then what is the value of \( F \) when \( y = 600 \)?

45. Below is the cost data of a toy manufacturing company.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Cost per Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>First 1000 units</td>
<td>$5</td>
</tr>
<tr>
<td>Next 1000 units</td>
<td>$2</td>
</tr>
<tr>
<td>Further units</td>
<td>$4</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>$1,500</td>
</tr>
</tbody>
</table>

a) What is the total production cost of 5,000 units?
b) What is the variable cost for producing 5,000 units

c) If each unit can be sold for $48.95, what is the total revenue for 5,000 units?

46. Jennifer operates an office supplies store. She ordered two times as many boxes of ballpoint pens as boxes of felt-tip pens. Ballpoint pens cost $3.50 per box, and felt-tip pens cost $4.50. If Jennifer’s order of pens totaled $138, how many boxes of each type of pen did she buy?

47. JB Cards Store spent a total of $950 ordering 600 cards from Hammer Company. The kids’ cards cost $1.75 each and nature’s cards cost $1.50 each. How many of each card did the store order?

48. Francis distributes the Toronto Star newspaper to a certain number of houses in northwest Brampton area. He charges $4.00 per week and collects the money every Saturday. In September, Francis had $72 in his wallet before he started collecting $4 from each house. When he returned home, he found that he had $206 in his wallet but he had forgotten the number of customers who had paid him. How can he find that number?

49. Translate each of the following algebraic expressions back into words:
   a) 2C – 4   b) 17 -2x   c) 6(b-9)   d) 4n + 1   e) n + 2   f) n + (n+1)
   g) 3x + 2   h) 17 + y   i) 2g - 4

50. A wholesale cost of an executive chair is $375, and that of secretarial desk is $300. Allan Furniture Company ordered 40 chairs for $12,825. How many desks of each type were ordered?
51. Tickets for a school dance are sold at $4 for adults and $2 for children.
   a) If \( p \) adults and \( q \) children buy tickets, write a formula for the total value, \( T \), of the ticket sales.
   b) Find the total value of the ticket sales if \( p = 50 \) and \( q = 20 \).

52. The equation or formula \( C = 0.04t + 10 \) represents the relationship between total the cost, \( C \), charged by an internet provider and time, \( t \), spent on the internet.
   Evaluate the formula when \( t = 2.5 \) hours \( C = \$40 \)

53. A computer decreases in value over time. The relationship between the value of the computer, \( V \), in dollars after \( t \) years, is written as an equation, \( V = -300t + 2100 \)
   Evaluate the equation when a) \( t = 5 \), b) \( t = 3.5 \) c) \( t = 3 \)

54. Beautiful Princess banquet hall charges a fixed rate plus a cost of $15.00 per person. For a party of 50 people, the total cost is $1200. Using \( C \) to represent the total cost, in dollars, and \( n \) is the number of people attending, write a formula for this relationship.

55. If \( x = 2 \), what is the value of \( 2x + 5 \)?

56. The cost, \( C \), in dollars to print \( n \) leaflets is given by the formula,
   \[ C = 35 + 0.03n \]. What is the cost of printing 900 leaflets?

57. Solve these equations.
   a) \( 8c + 18 = 114 \)   b) \( 17t - 1343 = 0 \)   c) \( \frac{x}{12} = 48 \)   d) \( \frac{t}{6} + 9 = 13 \)
   e) \( 10 + 2x = 15 \)   f) \( 3(a + 2) = 21 \)   g) \( 6x + 7 = 73 \)   h) \( 3x + 2 = 19 \)
58. Complete the table below.

<table>
<thead>
<tr>
<th>English Phrases</th>
<th>Algebraic Expressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Four increased by twice a number</td>
<td>$4x + 2$</td>
</tr>
<tr>
<td>Ten times the difference of a number and 14</td>
<td>$5(n+2) - 3$</td>
</tr>
<tr>
<td>Six less than two-thirds of a number</td>
<td>$2y + 5$</td>
</tr>
<tr>
<td>Fifty subtracted from three times a number</td>
<td>$\frac{1}{2}n + 8$</td>
</tr>
<tr>
<td>Three-fourths of the sum of a number and 12</td>
<td></td>
</tr>
</tbody>
</table>

59. The charges on a monthly water bill are $0.86 per m³ of water used plus a service charge of $4.49. Let $C =$ total charge, in dollars, and $W =$ total amount of water used, m³. Write a formula or equation to represent the relationship between $C$ and $W$.

60. An apartment company rents one-bedroom for $625 and two-bedroom apartment for $900. A total of 14 apartments rent for $11,225 a month. How many of each type of apartments does the company have?

61. The value in cents of $q$ quarters and $d$ dimes is given by the formula,
   $$V = 25q + 10d,$$
   where $q =$ number of quarters; $d =$ number of dimes.
   If the total value of $15.05 includes 58 dimes, find the number of quarters.
62. Cecilia needs to rent a car. She considers the following price equations where $C$ is the total cost in dollars and $d$ is the number of days.

<table>
<thead>
<tr>
<th>Company</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rent-way</td>
<td>$C = 20d + 100$</td>
</tr>
<tr>
<td>Cheapies</td>
<td>$C = 25d + 50$</td>
</tr>
<tr>
<td>Reasonable Cars</td>
<td>$C = 50d$</td>
</tr>
<tr>
<td>Drive Away</td>
<td>$C = 15d + 125$</td>
</tr>
<tr>
<td>Super Rent</td>
<td>$C = 5d + 200$</td>
</tr>
</tbody>
</table>

a) How much will it cost to rent a car from each company for 5 days?

b) Which company should she choose if she is planning to rent the car for at least 10 days?

63. Nicky compares the prices to download music from two different websites:

<table>
<thead>
<tr>
<th>New Tunes</th>
<th>New Music</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5.00 sign up fee</td>
<td>$9.00 sign up fee.</td>
</tr>
<tr>
<td>$0.75 per song</td>
<td>$0.50 per song</td>
</tr>
<tr>
<td>Hurry and get the latest song!!!</td>
<td>Make your day with music.</td>
</tr>
</tbody>
</table>

Nicky wrote the equation below to determine when the two plans will cost the same. She uses $m$ to represent the number of songs.

$$0.75m + 5 = 0.50m + 9$$

a) How much does it cost to download 15 songs with the two plans?
b) What number of songs must be downloaded for both music plans to cost the same?
c) If the New Tunes company were to gives $1.00 off after downloading 20 songs will both plan cost the same? Explain your answer.

64. What number would you add to the following expressions to give $f$ as the result?

a) $f - 1$  b) $f - 5$  c) $a - 17$  d) $f + 9$  e) $f + 12$  f) $f - 3$
65. Adding the same number to each side of an equation helps to solve certain equations, but suppose you had the equation, \( z + 18 = 36 \).

a) Can you add a whole number to the left side to give the result \( z \)?
b) What operation will change \( z + 18 \) to \( z \)?

66. Combine like terms in each of the following algebraic expressions.

a) \( 3x + 5x \)
b) \( 5x + x + y \)
c) \( 3a - 5a \)
d) \( 7b - 3b + 2b \)
e) \( 2mn - 5mn + mn \)
f) \( x + x + y + y \)
g) \( 6t + t - 4t + s \)

67. Solve each of the following equations.

a) \( 8y + 20 = 4 \)
b) \( 6x + 7 = 73 \)
c) \( 4y + 3 - y = y + 13 \)
d) \( 5a + 8 = 33 \)
e) \( 5a + a = 30 \)
f) \( 2(n + 1) = 14 \)
g) \( 3a - 5 = 10 \)
h) \( 5n = 45 \)

68. There are two major cell phone companies: Continental and Superden. They are offering two phone plans. Continental has \$20.00\ service charge per month and charges \$0.02\ per minute. Superden’s plan consists of \$5.00\ service charge and \$0.05\ per minute. At what point would the two phone plans be a better deal?

69. The table below shows Erica’s bank account balance for the past six weeks.

<table>
<thead>
<tr>
<th>Week (w)</th>
<th>Balance (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$3,000</td>
</tr>
<tr>
<td>2</td>
<td>$3,240</td>
</tr>
<tr>
<td>3</td>
<td>$3,380</td>
</tr>
<tr>
<td>4</td>
<td>$3,520</td>
</tr>
<tr>
<td>5</td>
<td>$3,660</td>
</tr>
<tr>
<td>6</td>
<td>$3,800</td>
</tr>
</tbody>
</table>

a) Using the information from the table, write an equation or formula that represents the balance, \( b \), in Jennifer’s account in relation to the number of weeks.
b) Use your equation or formula to find what Jennifer’s bank account balance will be in week 8.
70. The total fare for 2 adults and 3 children for a circus is $14.00. If a child’s fare is half of an adult fare, what is the adult fare?

71. The table below shows the number of security guards, \( g \), needed for a certain number of students, \( s \), at a college Esther dance.

<table>
<thead>
<tr>
<th>Number of Students (s)</th>
<th>32</th>
<th>48</th>
<th>80</th>
<th>272</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Security (g)</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>12</td>
</tr>
</tbody>
</table>

a) Write a formula or an equation that represents the relationship between the number of security guards needed and the number of students attending the dance.

b) How many security guards will be needed for a dance that has 24 students?

72. The formula, \( 8m + 5 \) represents the total cost in dollars, including shipping, for a certain number of music CDs, \( m \). What is the total cost for 5 music CDs?

73. Explain why \( 12 \times t = 9.6 \) and \( t = \frac{9.6}{12} \) give the same result for \( t \).

74. BM Light bought 1000 light bulbs. Headlight bulbs cost $13.95 each and taillight bulbs cost $7.55 each. If BM Light spent $9,342 on light bulb stock, how many headlight bulbs and how many taillight bulbs did it get?

75. A telephone call from Brampton, Ontario, to any part of Canada between 6:00pm and midnight cost $0.40 for the first minute and $0.10 for each additional minute. Write an equation or formula for the relationship and calculate the cost of a call that lasts for a) 2 min b) 10min c) 25 min d) 35min
76. The monthly service charge on West End Community Association’s chequing account is $2.50 plus $0.25 for each cheque it writes. Find the monthly charge if the Association writes:
   a) 5 cheques       b) 10 cheques    c) 20 cheques    d) 21 cheques    e) 30 cheques

77. To print a universe magazine, it costs $675 to set up the type and additional $2.75 per copy.
   a) Write a formula for finding the cost of printing a copy of the magazine.
   b) Find the total cost for printing 10 copies, 300 copies, and 50 copies.

78. A real estate company bought promotional calendars and date books to give to its customer at the end of the year. The calendars cost $0.75 each and the date books cost $0.50 each. The company ordered a total of 500 promotional items and spent $300. How many of each item did she order?

79. A computer company sold 144 cases of two grades of computer papers. Cosfet paper cost $15.97 a case and standard paper cost $9.75 per case.
   a) If the store’s total sales were $1,715, how many cases of each type were sold.
   b) What was the dollar value of each type of paper the company sold?

80. ME is a retail chain with two branches: Matt and Zubes. The annual profit of the Matt branch is four times that of the zubes branch. ME’s total profit for the year is $1,350,000. Find each branch’s profit.

81. A company has 800 employees made up both females and males. There are 20 more females than males. How many of the employees are females?
82. The combined daily sales of two branches of a major retailer, A and B, are $60,000. The daily sales of A is $2,000 more than that of the B. How much is the daily sales of each branch?

83. What will you subtract from each side to solve for k?
   a) 0 = 13 + k   b) -0.8 = k + 33   c) 19 = 19 + k

84. What do you have to multiply each side of the following equations in order to solve for p?
   a) \( \frac{P}{8} = -4 \)   b) \( \frac{P}{11} = -3 \)   c) \( \frac{P}{12} = -2.0 \)   d) \( \frac{P}{q} = r \)

85. By what do you have to divide each side of the equation below to solve for y?
   a) 14y = 128   b) 45 = -9y   c) 0.8 = -1.6y   d) xy = z

86. What do you have to add to each of the following equation to solve for e?
   a) e – 15 = 2   b) 141 = e – 8   c) e – 14.5 = 20.8   d) e – x = y

87. Simplify each.
   a) \( m^3 \times m^3 \times m^3 \)   b) \( y^3 \times x^2 \times y^2 \times x \)   c) \( s^3 \times s \)   d) \( a^2 \times b^2 \)

88. Simplify by combining like terms.
   a) 2k + 3k – 6k + k   b) 4t + 2t – 3t   c) 13y – y + x   d) 24v + v + w

89. In an office building, there are 8 floors and the number of rooms in each floor is \( y \). If each room has exactly \( c \) chairs, write an algebraic expression for the total number of chairs in the building.
90. Jane took a taxi to go to work. The distance between her house and work is 23 kilometres. The taxi charges consisted of $2.75 plus $1.57 per kilometers travelled.

a) Write an equation relating the total cost, \( C \), in dollars, to \( k \), kilometers travelled.

b) Use your equation to calculate how much the taxi charged Jane.

c) How much change did Jane receive if she gave the taxi driver a fifty dollar bill? (Answer correct to the nearest 5 cents).

d) How much was left out of the fifty-dollar bill after Jane gave the driver a tip of $5?

91. The following are the price quotes of two businesses that rent boats at a resort Centre.

<table>
<thead>
<tr>
<th>Maker Boats</th>
<th>Sonic Boats</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rent Charges</td>
<td>Rent charges</td>
</tr>
<tr>
<td>$15 First hour</td>
<td>$12 per hour (or part thereof)</td>
</tr>
<tr>
<td>then $10 per hour (thereof)</td>
<td></td>
</tr>
</tbody>
</table>

Ray rented a boat from Maker Boats from 8:15 am to 4:30 pm.

a) How long did Ray rent the boat? Give your answer in hours and minutes.

b) Complete the table below for the cost to rent a boat from Sonic Boats

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost to rent($)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c) Dora argued that if Ray had rented the boat from Sonic Boats, it would have been cheaper. Do you agree with Dora? Explain your answer with appropriate calculations.

92. The cost, \( C \), in dollars of buying an office condominium on level \( L \) of a building in downtown Toronto is given by

\[
C = 100,000 + 5000 \ (L - 33) \text{ if the level is above the 33rd floor.}
\]

Find the cost of an office condominium on level 42.
93. a) What is the meaning of a break-even point? Explain using your own example.
   b) What is the difference between fixed cost and variable cost?

94. If fixed cost is $50 and variable cost is $20, then the cost of
   
   1 item is $20 + $50 = $70
   3 items is $60 + $50 = $110
   20 items is $400 + $50 = $450
   
   What is the cost of x items?

95. a) Simplify \( \frac{1}{(4t)^2} \)
     
     b) Simplify by combining like terms: \( x^2 + x^2 + x^2 + x^2 \)

96. Given that \( v^2 = u^2 + 3as \), calculate the value of \( v \) when \( u = -6, a = 5 \) and \( s = 0.8 \)

97. The cost of renting a car is calculated using the formula
   
   \[ \text{Cost} = 20d + \frac{12(m - 50d)}{100}, \]
   
   where \( d \) is the number of days the car is rented and \( m \) is the number of kilometers the car is driven. A car is rented for 7 days and driven for 497 kilometres. Calculate the cost of renting the car using the above formula.

98. A manufacturing company has the following data relating its operations.
   
   Per machine: Material cost $8 Variable selling cost $3
   Labour cost $9 Selling price cost $30
   
   Fixed expenses are $20,000. Calculate a) The break-even point in quantity and in dollars and b) The profit if 4,500 machines are sold.
99. Simplify each by collecting like terms.
   a) $x + 5x$     b) $3x + 6x - 3x$     c) $2x + 7y - 4x + 3y + 1$     d) $5a + 6a - 4$

100. This month Sarah worked six hours less than twice the number of hours, $h$, he worked last month. Write an algebraic expression to represent this situation.

101. Sandy wants to buy a new 52 inch plasma television set from Rock Electronic Store. All new television sets at the store are $\frac{3}{4}$ of the original selling price, $p$. She has a $50 discount coupon she will use to buy the television. Write an equation that Sandy can use to calculate the final price, $f$, of the television.

102. a) Find the value of $7x + 2y$, when $x = 4$ and $y = 3$
   
   b) Find the value of $4x^2 - 5$, when $x = 3$

103. Mark is investigating the cost of using a cell phone. He finds two phone plans.

<table>
<thead>
<tr>
<th>Plusta</th>
<th>Modern</th>
</tr>
</thead>
<tbody>
<tr>
<td>*$14/month fixed charge.</td>
<td></td>
</tr>
<tr>
<td>*100min of free calls</td>
<td></td>
</tr>
<tr>
<td>* Then10cents per min thereafter.</td>
<td></td>
</tr>
<tr>
<td>Pay as you use</td>
<td></td>
</tr>
<tr>
<td>No fixed charge</td>
<td></td>
</tr>
<tr>
<td>No free calls minutes</td>
<td></td>
</tr>
<tr>
<td>$0.10 per minutes</td>
<td></td>
</tr>
</tbody>
</table>

Mark expects to use 200 minutes a month of phone calls. Which plan is cheaper? Explain your answer with suitable calculations.

104. A neighbourhood basketball team is planning to fundraise to buy some jerseys and balls. The team needs $450 for set up costs to sell hot dogs. The set-up costs include the purchase of hot dogs, buns, condiments, and grill equipment. It plans to sell each hot dog for $0.50.

   a) Calculate the revenue if 900 hot dogs are sold.
b) How many hot dogs should the team sell in order to break-even?

c) The profit formula is \( y = 0.50x - 450 \). Explain the meaning of the 0.50 and the 450 in the formula.

d) Use the profit formula to calculate how much profit the team can make by selling 1000 hot dogs.

e) If the team purchased 1500 hot dogs, what is the largest profit can it make?

f) Should the team reconsider its decision to sell hot dogs for fund raising if it is able to sell only 950 hot dogs? Why?

g) Explain if $0.50 is a reasonable price to charge for a hot dog.

105. Combine the like terms. a) \( x + 4x \)  b) \( 3x + 5x - 2x \)  c) \( 2x + 7y - 4x + 3y + 1 \)

106. Julie wants to attend a carnival. The admission fee is $8. Tickets for rides cost $4 each. Julie needs one ticket for each ride.

a) Write an equation that Julie can use to determine the number of ride tickets, \( r \), she can buy if he has only $32 for the rides.

b) Write another formula that can be used to calculate the total cost of attending the carnival.

107. Find the value of \( 5p + 2q \), when \( p = 4 \) and \( q = -7 \)

108. Solve and check.  a) \( 2(x + 3) = 8 \)  b) \( 4 = 3(2x - 5) \)

109. Juliano’s Towing Company charges $40 to hook a vehicle to the tow truck and $1.80 for each kilometer the vehicle is towed. Write an equation to represent the relationship between the number of kilometres towed, \( k \), and the total changes, \( C \).
5.1 Definition of Ratio

Ratio is a quantitative comparison of the relative size of two or more quantities with the same units of measurement. Businesses use ratios in a variety of situations and it is a powerful tool for making financial decisions. It is particularly important for analyzing business profitability and its ability to meet debt payment. As well, knowledge of ratio helps us make all kinds of comparisons in business. A common ratio in business is to compare the value of current assets to that of current liabilities. Another common ratio is to compare earnings after taxes to equity. Sometimes businesses compare the cost of raw materials to cost of labour. As well, in the manufacturing of confectionary products such as bread, candies and muffins, there are standard proportion of ingredients that each product should contain.

5.2 Properties of Ratio

The terms of a ratio are the numbers in the ratio. No term of a ratio can be zero. For a ratio in standard form, all the terms are integers and there is no common divisor for all the terms. For a ratio with more than two terms, it is possible for two or more of the terms to have a common divisor but not for all of the terms. A ratio can always be expressed in standard form by multiplying and/or dividing each of the terms by the same number. Multiplying or dividing each of the term of a ratio by the same number except zero, do not change terms of a ratio, because they are relative comparison of their sizes.

**Notation** Ratio of quantity A to quantity B is written in mathematics as follows, quantity A: quantity B. This is read as follows: A is to B, \( \frac{A}{B} \) or A to B. A is called the first term and B the second term of the ratio. Therefore, A: B is called a two-term ratio. A: B: C is called a three-term ratio. Terms of a ratio can be reduced to lowest term just as we reduce fractions into lowest term.
Example 1:
The daily sale at Bagra Enterprises is $1 500 and at Akua Enterprises $2 000. The ratio of daily sales at Bagra to daily sales at Akua Enterprises is written as;
Daily Sales at Bagra Enterprises : Daily Sales at Akua Enterprises = $1 500 : $2 000. Each term can be divided by $500 (or by 100 and then by 5) to reduce them to lowest term:
Daily Sales at Bagra Enterprises : Daily Sales at Akua Enterprises = (1500 ÷ 500) : (2000 ÷ 500) = 3:4. (What does this mean?)

Interpretation: 1. For every 3 dollars in daily sale at Bagra, there is $4 in sale at Akua. 2. If the total daily sales for Bagra and Akua are divided into 7 (which is 3 + 4) equal parts, 3 of the parts are Bagra’s sales and 4 parts are Akua’s sales.

5.3 Finding Ratios of Quantities

Example 2
The hourly wage of a sales associate is $9.50 at Balmort; $10.50 at Scheers; and $12 at Makola. Find the ratio of the hourly wage of sale associates at Balmot to that of Scheers and to Makola.

Solution
Balmot : Scheers : Makola = $9.50 : $10.50 : $12 then (Multiply each term by 100 and divide by $) = 950 : 1050 : 120 (Divide each term by 50, which the common factor)
Balmot: Scheers: Makola = 19 : 21 : 24

Example 3
The dimensions of a rectangular box are length 1m 35cm, width 1m 80cm, and depth 90cm. Find the ratio of the length to the width to the depth of the box.

Solution
Length: Width : Depth = 1m 35cm : 1m 80cm : 90cm then (Change units to cm = 135cm: 180cm: 90cm then (Divide each term by 45cm)= 3 : 4 : 2
Example 4

The ratio of 4:2 is not in standard form but 3: 4: 2 is in standard form. The ratio 4 : 2 is expressed in standard form by dividing both terms by the common divisor 2; Thus, 4 : 2 = 2 : 1

For a general ratio;  A : B : C : D = kA : kB : kC : kD  k any number except zero.

=  \frac{A}{m} :  \frac{B}{m} :  \frac{C}{m} :  \frac{D}{m}  m any number except zero.

A change in the order of a ratio changes the ratio. The ratio of quantity A to quantity B is not the same as the ratio of quantity B to quantity A.  A:B \neq B:A

Example 5

This example shows that we can use our understanding of ratio to calculate unknown quantity. This idea is very important to understanding proportion. It should be stated that proportion is an extension of ratio.

Joe’s two-week wages is $800 for 80 hours of work.

a) Given this ratio, how many hours does he have to work in order to earn $1600?
b) If he works 245 hours how much will he earn?

a) Given the ratio,  \frac{800}{80} = 10:1  (After diving each term by 80)

\[ \downarrow \quad \downarrow \]

Wages : Hours

\[ \frac{1600}{h} \]

Since for every $10, he has to work an hour, there are 160 tens in $1600. So, Joe has to work 160 hours in order to make $1600. Note that the ratio of wages to hours does not change, except when one of the terms of the original ratio changes.

b) Again, he has to work 240 hour and $\frac{5}{10}$ of an hour; that is 30mins.

This example seems very simple and it should provide you an insight into the nature of ratios in mathematics. However, other examples may not be so simple and they may require a more sophisticated procedure.
5.3 Comparing Ratios

An important idea in mathematics is how to compare ratios. It gives us an idea about the multiplicative relationship between quantities. For example, what is the relationship between 1/2 and 2/4? Do you think that 1/2 and 2/4 equivalent fractions or ratios? With a little drawing and intuition, it becomes clear that 1/2 and 2/4 are equivalent fractions or ratios. And that 1/5 and 20/100 are also equivalent. Ratios are equivalent or equal if their cross-products are the same, or if they can be written as equal fractions. To compare ratios do the following:

a) Write the ratios as fractions
b) Calculate their cross-products
c) Then compare them. If the products are the same, the ratios are equal or equivalent.

Example 5
Are the ratios 3 to 4 and 6/8 equal? The ratios are equal if \( \frac{3}{4} = \frac{6}{8} \). Their cross-products of \( 3 \times 8 = 6 \times 4 \) are obviously equal. Since both products are equal, the ratios are equal.

Example 6
Are the ratios 1:5 and 4/20 equivalent? Yes, they are equivalent. The fraction 4/20 can be reduced to the lowest by dividing the numerator by 4 (\( 4 \div 4 = 1 \)) and the denominator by 4 (\( 20 \div 4 = 5 \)). This gives us 1/5, which is equal to 1/5. Their cross-products give, \( 1 \times 20 = 5 \times 4 \).

Example 7
Compare 13:17 and 5/7. Are they equal or equivalent ratios? No, they are not equivalent because their cross-products are not equal: \( 13 \times 7 \neq 17 \times 5 \).

5.4 Proportional Division

This is the division of a quantity in a given ratio. That is, the given quantity is divided into piles such that the ratio of quantities in the piles is equal to the given ratio.

Example 1
Maya and Nina’s uncle gave them an amount of $150,000 in his will. The money is to be divided between them in the ratio of 3 to 2. How much does each receive?
Solution

Since when the $150,000 is divided between them it should correspond to the given ratio, we can try a few pairs of numbers.

Maya’s share : Nina’s share

$100,000 : 50,000

\[ \frac{100,000}{50,000} = 2 : 1 \]

After dividing each term by 50,000, you can see that

\[ \frac{2}{1} \]

does not correspond to 3:2. So, we try another pair:

\[ \frac{90,000}{60,000} = 3 : 2 \]

After dividing each term by 30,000

Thus, Maya’s share is $90,000 and Nina’s share is $60,000. This is correct because the divided quantity corresponds to the given ratio.

Example 2

Mohammed, Ben, and Rachel earned some money by selling toys and lemonade at a festival stand. They agreed to share any profit in the ratio of their ages. Mohammed is 9 years old, Rachel is 10 years old, and Ben is 7 years old. Ben’s share came to $28. Find the share of Mohammed and that of Rachel.

Solution

The solution to this problem is not so easy, as example one. This is because the amount of profit they made is not given. First, let us find the profit they made.

Let \( p \) stand for the profit they made. The total ratio is 26 (9 + 10 + 7).

This is how Ben’s share was obtained:

\[ \frac{7}{26} \times p = 28 \]

\[ \frac{7p}{26} = 2 \]

\[ 26 \times \frac{7p}{26} = 28 \times 26 \]

We multiply each side by 26 in order to clear the fractions.

We now have 7p = 728

The 728 was obtained after multiplying 28 by 26.

P = 104

After dividing each side of the equation by 7.
The profit was $104.00

Mohammed’s share = \( \frac{9}{26} \times 104 = $36 \)

Rachel’s share = \( \frac{10}{26} \times 104 = $40 \)

The ratio is 36:40:28

= 9:10:7 after dividing each term by 4.

5.5 Review, Exercises and Assignments

1. Give two examples of situations in which a business owner may use ratio for the purpose of comparison.

2. Reduce to the lowest terms
   a) 12 to 24   b) 84 to 56   c) 15 to 24 to 39   d) 32:100   e) 30:240   f) 50:49   g) 10.5:3.5

3. Set up a ratio for each of the following and reduce to lowest terms.
   a) 12 dimes to 5 quarters   b) 6 minutes for 50 metres   c) $45 per day for 12 employees for 20 day   d) $2000 sales for 5 days

4. The cost of a unit of a product consists of $4.25 direct materials cost, $2.75 direct labour cost, and $3.25 overhead cost. What is the ratio existing between these three elements of cost?

5. Mariam applied for residential mortgage from Seaway Financial Services. To assess her eligibility for the mortgage, the financial officer used the following ratio:
   Personal debts: personal annual income.
   a) Mariam’s personal debts were $40,000 and personal annual income was $50,500. What is her ratio of personal debt to personal annual income?
   b) If you were the financial officer, would you approve the mortgage for Mariam? Why?

6. a) The gross sales to the number of sales persons ratio for a retail company is $20,000:5 for the first quarter. What does this mean to the retailer? What advice would you give to the retailer, if the total wage bill for the quarter is $15,000?
   b) The ratio of sales to salaries expenses for a grocery store is 600:1 for the third quarter. Explain what this means for the retail operations.
7. Mike received $100 a week for distributing flyers to houses. He spent $40 at the movies and $25 on comic books. After buying $10 worth of ice cream, he saved the rest.
   a) What is the ratio of the amount of money he spent on buying comic books and movies to the amount he saved?
   b) What does the ratio mean?

8. The table below shows the net profit in 2000, 2001, and 2002 of three firms in the bakery industry.

<table>
<thead>
<tr>
<th>Firm Name</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multi Bakery Ltd</td>
<td>$200,000</td>
<td>$300,000</td>
<td>$120,000</td>
</tr>
<tr>
<td>All Light Ltd</td>
<td>$100,000</td>
<td>$125,000</td>
<td>$150,000</td>
</tr>
<tr>
<td>Vim Bakery Ltd</td>
<td>$250,000</td>
<td>$100,000</td>
<td>$50,000</td>
</tr>
</tbody>
</table>

   b) Which company or companies do you think did the better over the three years in terms of net profit performance? (Use the ratios you calculated in (a) to support your explanation. Do not for get to disclose any assumptions you will make in your explanation.).
   c) Why do you think profit ratios help us to compare the three companies?

9. Some banks calculate the ratio of loans to deposits in order to measure how much loans they have given out in terms of deposits. This ratio is important because a bank does not want to face the risk of running out of money in case its customers start withdrawing more of their deposits than expected. The ratio is loans: deposits, or loans/deposits. ABC bank gave out $1,800,000 loans to its customers and took deposits of $4,000,000 in the first two quarters.

   a) Calculate the bank’s loans to deposits ratio for that period.
   b) What does the ratio mean for the bank?

10. The ratio of ten-dollar notes to twenty-dollar notes in a cashier’s till is 4:5. There are 73 notes altogether. What is the value of the 73 notes?
11. Arctic Transportation Ltd and R&T Transportation Ltd have different pricing policies. For Arctic, one route adult fare is $1.20 and kid fare is $0.40. For R&T Transportation on different route, the adult fare is $1.40 and kid fare is $0.70.
   a) Find the ratio of child fare to the adult fare for each company.
   b) Which company gives kids the better deal?

12. Determine which items in the following pairs is the better buy. Make a table showing unit pricing.
   a) 900 g can of peaches for 49 cents or an 850g can for 46 cents.
   b) A giant-size box 1500 g of detergent for $11.51 or a king-size box 2400 g for $12.47.
   c) 798 ml of baby cream for $6.97 or 444ml for $3.76.

13. Write the reduced fractions that represent the following ratios.
   a) The kilometer-per-litre ratio for a trip of 387 kilometres on 40 litres of gas.
   b) The student-to-faculty ratio of a community college at a community college with 5000 students and 150 faculty.
   c) The administrator to employee ratio if there are 15 administrators and 850 employees.
   d) The sales commission-per-salesperson of $12000 for 9 sales persons.
   e) Profit for 3 baskets of apples is $12.

14. Western Technology has total liabilities of $150,000 and owner’s equity of $75,000.
   a) What is the ratio of total liabilities to equity?
   b) What does the ratio you calculated in a) mean?
   c) If you were a bank manager, would you give credit to this company? Why?

15. Ms. Taylor owns Baby Centre which has current liabilities of $100,000 and current assets of $150,000. Ms. Taylor did the following calculation:
   $150,000 − $100,000 = 50,000. She then concluded that if she were to pay off her current liabilities, she would be left with $50,000.
   a) Calculate the ratio of her current liabilities to current assets.
   b) Is the ratio more useful information than Ms. Taylor’s calculation? Explain.

16. A city council’s rules require that the ratio of care provider to children is 1:4.
   a) How many care-providers are needed for a day care with 52 children?
   b) How many children are there in a daycare with 20 care-providers?
   c) A day care with 24 children has only six care-providers. Does this fulfill the regulation? Explain your answer.
17. A storekeeper orders two hundred 60g pack of roasted peanuts. A 60g pack of peanut sells for $1.40. Calculate the price of a 100g pack of peanut at the same price per gram.

18. A recipe for lemonade requires 8 lemons and 1 cup of sugar.
   a) What is the ratio of lemon to sugar?
   b) If you want to make 27 litres of lemonade, how many lemon will you need?
   c) How many cups of sugar will you need for 63 litres of lemonade?

19. The manager of a music store estimates that the ratio of sales of cassettes to CDS is 4:7.
   a) In one day, 92 cassettes were sold. How many CDS were sold that day?
   b) On another day, 84 CDS were sold. How many cassettes were sold that day?

20. A grocery store sells potatoes by the kilograms. It sells 5 kg of potatoes for $1.20. Find the cost of
   a) 7kg of potatoes  b) 20kg of potatoes  c) 2 kg of potatoes  d) 10kg of potatoes

21. A Meto share currently sells for $20.00 on the stock exchange. For the previous twelve months, the earning per share is $1.25. What is the ratio of price-to-earning?

22. A central library in a small town has 720 non-fiction books and 400 fiction books. The town has population of 4,200.
   a) Find the ratio of fiction books to non-fiction books.
   b) Find the new ratio if 40 new fiction books are purchased for the library.
   c) What is the population to library books ratio after the new purchase?
   d) What is the usefulness of the ratio you calculated in c)?

23. A car parking lot contains 400 parking spaces. Of these spaces, 60 are short-term and the rest are long-term. The fee for short-term spaces is $3 per hour and the fee for long-term spaces are $14 per 24 hour day:

   a) Find the ratio of the short-term spaces to the long-term spaces.

   b) If the short-term spaces are full an average of 8 hours per day, find the ratio of the revenue gained from short-term parking spots to long-term parking spots.
24. The data below shows the cash position of two companies in the same industry.

<table>
<thead>
<tr>
<th></th>
<th>Tanko Ltd</th>
<th>Supreme Ltd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash to asset ratio</td>
<td>2:3</td>
<td>1:3</td>
</tr>
</tbody>
</table>

a) Interpret the ratios
b) Explain why cash to asset ratio is important to businesses?
c) Which company is likely to face a cash crisis in future? Why?
d) If Tanko has cash of $2,500,000, what is its total assets?

25. In a certain university, 4 out of 24 students belong to student clubs. What is the ratio of student in student clubs to students not in student clubs?

26. Amena and Tina received 63 text messages in the ratio of 5 :4. How many text messages did Tina receive?

27. The ratio 35,000,000 : 50 can be written in the form n: 1. What is the value of n? How was 1 obtained? Explain your answer.

28. A resort retail outlet employs a manager, sales clerk, and a part-time administrative assistant. The yearly bonus is distributed in the ratio 6: 4: 3 for the manager, sales clerk and the part-time administrative assistant respectively. The total bonus for 2007 is expected to be over $65,000. What is the expected minimum bonus for each?

29. Yen, Casandra, and Muthali own a fashion accessories store. Their agreement is to share net profit from the store in the ratio of the time each spend in the store for the year. Yen spends twice as much time as Casandra, and Casandra spends thrice as much time as Muthali spends in the store. The net profit from the store was $120,000. How much will each receive?

30. A computer retail store employs a manager, two sales associates, and cashier. The ratio of the manager’s salary to the sales associates is five to three, while the ratio of the sales associates’ salary to the cashier’s is four to three. The two sales associates are paid equally. The store spends one hundred fifty thousand dollars on salaries a year. What is the salary of each employee?

31. Give an example of how a ratio can be used for the purpose of comparison.

32. A, B and C make, in $ per hour, 5, 7, 11 respectively, owing to experience. They all worked together to complete a similar job and earned $2,580. What is C’s share of the earning?
33. Tanya went fishing with her little sister. They caught 30 fish but 2 out of every 3 were too small to take home. How many fish were too small to take home?

34. An amount of $55,000 is be shared amongst three friend, A, B, and C in the ratio 3:5:3. How much does A receive?

35. The total cost of producing 1000 serton medication is $125. The ratio of material to labour for producing that quantity of serton is 3:1. Variable overhead expense is $20 and fixed expense is $5.00.
   a) What is the material cost of producing 1000 serton?
   b) How much does it cost in total to produce 100 serton?

36. A packet contain only frozen red and green beans of equal mass in the ratio of 4:5. If there are 350g of green beans in this packet, what is the total mass of the packet?

37. A retail store has two departments: Meat and grocery. Fixed expenses of the store for the month is $900. This must be shared between the two departments in the ratio of 4:5. What is the grocery department’s share of the fixed cost for the month?

38. Find the value of each variable in the following sets equivalent of ratios.
   a) \( 3: 20 = a: 80 = 24: k \)  
   b) \( 7:18 = m: 6 \)  
   c) \( \frac{7}{b} = \frac{49}{21} = \frac{f}{42} \)

39. The prices of two jeans are in the ratio of 11:13. If the less expensive jeans costs $24.50, what is the price of the other?

40. Use colon notation to write the following as ratios, in lowest terms.
   a) 16mm, 64 mm  
   b) 500kg, 2,500 g  
   c) 70cm, 700m

41. The ratio of cost price to selling price of an article is 3.5:6.5. What is cost price of the article if its selling price is $10.40?

42. State two ratios equivalent to 1:5.

43. Sue and Alex agreed to share profits from their partnership in the ratio of 1:2. If Alex receives $9180, how much does Sue receive?
44. The waist measurement and length a woman’s skirt are in the ratio 1.4:1 If Ann’s waist measurement is 70cm, what is the length of her skirt?

45. Reduce the following ratios in lowest terms.
   a) 18: 72  
   b) \( \frac{330}{1100} \)  
   c) \( \frac{25}{100} \)  
   d) 3 : 1

46. What is the value of m when 5:6 = 25: m

47. The annual net profit of three companies, A, B and C, in the housing industry is in the ratio of 20:40 :120. Use the ratio concept to interpret the annual profit of the three companies.

48. The ratio of the annual income of the business faculty member to an English faculty member is 7:5. If the English faculty member’s income is $154,000, what is the income of business faculty member?
Percent symbol %, is the fraction \( \frac{1}{100} \), that is, \( \% = \frac{1}{100} \).

**Example:**

1. 50\% is also the fraction \( \frac{50}{100} \)
2. 127\% is also the fraction \( \frac{127}{100} \)
3. 12.5\% is also the fraction \( \frac{12.5}{100} = \frac{125}{1000} \)
4. 33\frac{1}{3}\% is also the fraction \( \frac{33\frac{1}{3}}{100} = \frac{100}{300} \)

Similarly, a given fraction has an equivalent percent, which is a ‘fraction’ with denominator 100.

**Example:**

1. \( \frac{3}{4} = \frac{3 \times 100}{100} = \frac{300}{100} = \frac{75}{100} = 75\% \) So \( \frac{3}{4} \) as a percentage is 75%.
2. \( \frac{5}{7} = \frac{5 \times 100}{100} = \frac{500}{100} = \frac{71\frac{1}{7}}{100} = 71\frac{1}{7}\% \) So \( \frac{5}{7} \) as a percentage is 71\frac{1}{7}%.
3. \( 1.32 = \frac{1.32 \times 100}{100} = \frac{132}{100} = 132\% \) So 1.32 as a percentage is 132%.
4. \( 5 = (5 \times 100)\% = 500\% \) So 5 as a percentage is 500%.

Any number \( C \) (whether is a fraction, decimal, or integer) as a percentage is \( (C \times 100)\% \).

**6.2 Decimal Form of Percent**

Percent can also be written in decimal form by dividing it by 100, which is the same as placing a decimal point two digits to the left.

**Example 3**

\( 5\% = 5 \div 100 = 0.05 \)

\( 14\% = 14 \div 100 = 0.14 \)

\( 20\% = 20 \div 100 = 0.20 \)

\( 35.2\% = 35.2 \div 100 = 0.352 \)

\( \frac{1}{2} \% = 0.50 \div 100 = 0.05 \)

\( 33 \frac{1}{3} \% = 33.33 \div 100 = 0.33 \)
6.3 Ratio form of Percent

Percent can also be written in ratio form with the percent being the first term and 100 the second term.

Example 4  
10% = 10:100; 54.5% = 54.5:100; 15% = 15:100; 75% = 75:100

6.4 Calculating Percentages

We should distinguish percent from percentage. Percent is the rate and percentage is the result when the rate is applied. Percent is part of a whole which is 100 rather than 1 in fraction sense. For example, 5% could be interpreted as 5 out of 100. So, when someone promises to give you 5% of his $200 pocket money, he simply means he will give you $10. That is, $5 out of $100 and $5 out of the other $100, making it $10 ($5 + $5 = $10). This is the same as 0.05 (5% changed to decimal) multiplied by 200.

Therefore, we have the following procedure for calculating percentage: percentage = rate x base. In our example, the $200 is the base, and the rate is 5%. Using letters (Do you remember algebra?), B= base, P= percentage and R= rate of percent, we can shorten the procedure to,

\[ P = BR, \text{ percentage is equal to the rate of percent multiplied by the base quantity.} \]

Thus, we have the following formulas to work with,

Percentage = Rate x Base, \( P = R \times B \), for finding the percentage

\[ \text{Base} = \frac{\text{Percentage}}{\text{Rate}}, \quad B = \frac{P}{R} \text{ for finding the base} \]

\[ \text{Rate} = \frac{\text{Percentage}}{\text{Base}}, \quad R = \frac{P}{B} \text{ for finding the rate.} \]

Therefore, to work with percent the student must be able to identify the rate and the base.
Example 1

In 2007, John bought a sofa chair for $299. He had to pay the GST of 6% and PST of 8% on his purchase. a) How much was the GST? B) How much was the PST? c) Will it make any difference if you were to apply a single rate of 14% sales tax?

Solution

a) GST = R x B
\[ = 299 \times 0.06 = $17.94 \]

b) PST = R x B
\[ = 299 \times 0.08 = $23.92 \]

c) No, it will not make any difference since \(0.14 \times 299 = $41.86\). This is the same as \(17.94 + 23.92\).

Example 2

Mary’s annual salary is $54,000. Her raise for this year is 3.5%. How much raise did she get?

Solution

Percentage (raise) = RB = 54000 \times 0.035 = $1890. She got a raise of $1890.

Example 3

An airplane was 85% full when it had 425 passengers. What is the full seating capacity of the airplane?

Solution

It should occur to you immediately you read the question that the airplane is full when it is 100%. This implies that a full-seating capacity occurs when it is 100% full.

Let y represent the full seating capacity of the airplane. Then
\[ 85\% \text{ of } y = 425 \]
\[ \frac{0.85}{0.85} \frac{y}{0.85} = \frac{425}{0.85} \quad \text{Divide each side by } 0.85 \]
\[ y = \frac{425}{0.85} = 500. \]
Alternatively, we could proceed to solve the problem as follows:

85% full = 425

1% full = 425 / 85 = 5

100% full = 5 x 100 = 500

The full seating capacity of the plane is 500.

The plane has a full-seating capacity of 500.

6.5 The Three-Step Method

We can use the three-step method to estimate or calculate percentages mentally. To use this procedure the student should be able to calculate 10% and 1% of a quantity with ease. And use the results to make approximations for percentages. This method involves three steps as the following examples illustrate.

Example 1

Francis bought a bicycle for her daughter costing $48.00. He had to pay sales tax of 14% on the $48. He was wondering an easy way to estimate the sales tax he had to pay. How should he go about this?

Solution

First, we have to figure this out: What is 10% of 48? (48 x 0.10 = 4.80, a matter of one decimal point to the left of 48)

Second, what is 1% of 48? (48 x 0.01 = 0.48, just two decimal places to the left of 48)

Third, since 10% of 48 is $4.80, we are left with finding 4% of 48. Halving 4.80 is 2.40, which is 5% of 48, but 5% > 4 %. We know that 1% of 48 is 0.48. So 2.40 – 0.48 = 1.92, that is 4% of 48. Therefore, 14% of 48 is 4.80 + 1.92 = 6.72.

The sales tax is $6.72.

If we were to do this mentally we would have just calculated 10% of 48, which is $4.80 and half this to get $2.40. This gives a total of $7.20, which is closer to $6.72.
What you have to bear in mind is that, in using the three-step method you have to take the easier route. For example, suppose you were calculating 6% of a quantity. First, calculate 10%, which is easier to do and half it to get a 5% of the quantity. You are then left with calculating 1% of the quantity. You may approximate this in any way you want. The key is to learn to calculate 10% and 1% of a quantity as fast as you can. After that you are ready to use the three-step method.

6.6 Percent increase and decrease

a) Finding Percent Rate of Increase or Decrease

Percent is often used to show either increase or decrease in amount. The price of lumber, for example, could increase from $2.40 a metre to $2.50. Hydro rate may jump from $0.0530 per kilowatt hours to $0.0570 per kilowatt hours. A paper manufacturing business may reduce the prices of its computer printing paper products from, say $3.97 per 500 sheets to $3.27. The increase or decrease in the amount can be conveniently expressed in terms of percentages. The following procedures are used.

Rate of change (Increased) = \( \frac{(NA - OA) \times 100}{OA} \), where NA = new amount and OA =Original amount.

Where NA is the new amount, and OA is the original amount.

Rate of change (Decrease) = \( \frac{(OA - NA) \times 100}{OA} \)

Example 1

Mary’s hourly wage was increased from $14/h to $15.50/h. What is the percentage? Increase in her wage rate?

Solution 1

The original amount is $14 and the new amount is $15.50. Therefore, this is an increase.
Change = \frac{(15.50 - 14) \times 100}{14} = 10.7 \Rightarrow \text{Mary’s hourly rate increased by about 11\%.}

**Example 2**

In a survey of 400 people, it was found that 240 watched *Everybody Loves Raymond*. What percent does not watch that show?

**Solution 1**

The original amount is 400. And those who watch that show are $240.

Change = (400 - 240) ÷ 400 x 100 = (160 ÷ 400) x 100 = 40\%

40\% of the people surveyed do not watch that show.

**b) Finding the rate of Percent Change**

Instead of finding the percentage change, sometimes we have a situation where we are given the percent and our task is to find the new quantity. In this case, we have two procedure options, both of which will give us the same ultimate answers. However, the procedure we should use depends on the situation.

1) We may calculate the percentage increase or decrease and add or subtract it from the original quantity. We have, Original Amount + increase, or Original Amount - decrease

2) We may calculate the new amount straight away by using the given percent.

Now let us have an illustration of both procedure options.

**Example 1**

What is the number when $40 is increased by 25\%?

**Option 1 (Calculating the percent increase or decrease separately)**

The original number is $40 and the change (increase) required is 25\% of $40.

\[40 + (0.25 \times 40) = ?\]

40 + 10 = 50. The new amount is $50.

**Option 2 (Calculating the new amount)**

The original quantity is $40 and the change (increase) is 25\% of $40.
The quantity $40$ represents 100% and must be increased by 25%. So the new quantity must be 125% (100 + 25).

Thus, \[ 40 : 100 \]
\[ N : 125 \]

By cross-product, \[ 100N = 40 \times 125 \]
\[ 100N = 5000 \] (Divide each side by 100)
\[ N = 50 \] (or \( 40 \times 1.25 \))

The new amount is 50.

**Example 2**

What is the amount due when a debt of $350 is reduced by 20%?

Option 1

The original amount is $350 and it is to be reduced by 20%.

\[ 350 - (20\% \text{ of } 350) = ? \]
\[ 350 - (0.20 \times 350) = ? \]
\[ 350 - 70 = $280 \]

Option 2

Since the original amount of $350 is to be reduced by 20%, 80% (100% - 20%) of the amount will be left. Therefore, \( 0.80 \times 350 = $280 \) is the new amount.

**Example 3**

After the price of a TV set was reduced by 15%, the sale price paid was $245.

a) What was the price of the TV set before the reduction? b) What was the amount of reduction?

Solution

a) We can use any of the procedural option to answer the questions. Note that we are required to find the original price of the TV.

Thus, \( 100\% - 15\% = 85\% \). Since the original amount was multiplied by 85% to get the new amount of $245, we should divide the new amount by 85% in order to get the original amount. \( 245 \div 0.85 = 288.24 \) is the original amount.
b) To find the amount of the reduction we may use one of these. \(288.24 \times 0.15 = 43.24\), or \(288.24 - 245 = 43.24\).

### 6.7 Commercial Discounts and Mark up

**i) Commercial Discount**

Businesses often give discounts to their customers. Discounts are reduction in the price of a product or service, which allows the customers to save money. Discounts are often quoted in percent that allows the purchaser to compare savings in terms of money. Sometimes a business may give series of discounts rather than one discount. There are many reasons a business offers discounts to its customers. Some of these reasons are stated below:

a) To reduce excessive inventories; It costs a lot of to carry inventories in the form of warehousing, insurance, and caretaking. By giving discounts to its customers, a business is able to reduce the amount of inventories it is holding.

b) To response to the prices of a competitor;

c) To encourage quick turnover or sell fast; and

d) To encourage consumer loyalty.

**Example 1**

A portable CD player costing $199.00 is on sale for $99. (a) How much is the discount? (b) What is the rate of discount?

**Solution**

a) List Price − Discount = sale price

\[199 - 99 = 100\]

The discount is $100

b) Discount rate = \(\frac{100}{199} \times 100 = 50.25\%\)

**Example 2**

Ahmed wants to buy a new carpet for his house. The new carpet costs $200. One day he saw the carpet being offered for 25% off the purchase price. (a) How much money does he save by buying the carpet? b) What is the net price?
Solution

a) Amount of discount = Rate of discount x List price
   \[= 0.25 \times 200 = 50\], Ahmed saves $50.00

b) Net price = List price – Discount amount
   \[= 200 – 50 = 150\]

Instead of calculating the amount of discount and then deducting it from the list, the net price or net cost can be calculated by using the more efficient formula developed in the illustration below.

In example 2, since the discount is given as a percent of the list price, the following is obvious:

List……………………$200 – 100% of List price
Less discount…………$50 – 25% of List price
Net price or Net cost     150 – 75% of List price

Note that the 75% is called the net cost factor or net factor. It is abbreviated as NCF and obtained by deducting 75% from 100%. That is, \(NCF = 100\% - 75\%\). We should let \(d\) represent discount and \(L\) be the List price. In general, this could be written as, \(NCF = (1.00 - d\%) \times L\) or
\[NCF = (1 - d) L, \text{ omitting the decimal point and the multiplication sign.}\]

Using this efficient procedure, the Net price in example 2 could be calculated as follows. \(NCF = (1 - 0.25)200 = 0.75 \times 200 = 150\)

In the case of series of discounts, that is more than one discount, the formula becomes, \(NCF = (1 - d_1)(1 - d_2)(1 - d_3…(1 - d_n)L.\) In this formula, \(d_1\) is the first discount, \(d_2\) is the second discount, \(d_3\), is the third discount and so on.

Example 3

*More Store Corner* is having a sale event. Ladies’ jeans pants regularly priced at $69 is being sold for 20 off and additional 10% on Saturdays only. Felicia bought 2 jeans on Saturday. How much does he pay for them?
Solution

NCF = (1 – 0.20) (1 – 0.10) 69
= (0.80)(0.90)69
= $49.68

In the above example, we can use rewrite the formula to calculate a single equivalent rate of discount, instead of applying series of discount to the base quantity. To rewrite the formula, we have to drop the L, so that we have,

\[ SDR = \{1 - (1 - d_1)(1 - d_2)(1 - d_3) \ldots (1 - d_n)\}. \]

Let’s use the above formula to test the accuracy of the answer in example 3.

\[ SDR = \{1 - (1-0.20)(1-0.10)\} \]
\[ = \{1 - (0.80)(0.90)\} \]
\[ = (1 - 0.72) \]
\[ = 0.28 \]

The single rate is 28%. If we apply it to $69, we get $19.32 which is the discount amount. To get the net price as follows: \[ NP = 69 - 19.32 = $49.68. \]

The most important thing about this formula is that you should be able to convert percents into decimals.

ii) Mark up

Businesses are set up for the primary purpose of making profit. So, businesses always add up their expenses, called overhead, and profit margin to the cost price of the goods or services they sell or produce in order to arrive at the selling price. The profit margin may be expressed as percent of the cost price of the goods/service or the selling price. The following relationship is therefore true,

Selling price = Cost of buying + Expenses + Profit

Using letters, it becomes, \[ SP = CP + E + P, \] where CP is the cost of buying the product or producing it, SP is the selling price, E is the expenses, and P is the profit.
Example 1

A retailer has some skirts that cost her $48 each. Her overhead expense for each shirt is $3. She wants to sell them at a profit of 15% of the cost. What price should she sell each skirt?

Solution

\[ SP = CP + E + P \]
\[ SP = 48 + 3 + 15\% \text{ of } 48 \]
\[ SP = 48 + 3 + 0.15 \times 48 \]
\[ SP = 48 + 3 + 7.20 \]
\[ SP = 58.20 \] She should sell each skirt at $58.20 in order to realize 15% profit on cost.

Example 2

A pair of shoes cost a retailer $32 and he sells each for $44.80, including overhead expenses. What is his rate of profit based on the cost?

Solution

\[ SP = 44.80, CP = 32, \]
\[ \text{Rate of Profit on cost} = \frac{(SP - CP) \times 100}{CP} \]
\[ = \frac{(44.80 - 32) \times 100}{32} \]
\[ = 40\% \text{ (approximately)} \]

The retailer’s rate of profit is about 40% on cost.

Example 3

A head of lettuce costs a retailer $0.45. (a) At what price should it be sold to make a profit of 40% on the selling price? (b) What is the actual profit on each lettuce head?

Solution

a) Cost price = $0.45, SP =? P = 40% on selling price

\[ SP = CP + E + P \]
\[ SP = 0.45 + 0 + 0.40(SP) \]
\[ SP = 0.45 + 0.40SP \]
\[ SP - 0.40SP = 0.45 \]
\[ 0.60SP = 0.45 \text{ (divide both sides by 0.60)} \]
SP = \frac{0.45}{0.60} = $75.00

b) \ P = 40\% \ of \ $75 = 0.40 \times 75 = $0.30. \ The \ retail’s \ profit \ is \ $0.30.

6.8 Special Pricing Consideration for Produce

Grocery stores that sell perishable produce such as orange, kiwi, apples, pear, honeydew, pineapple, potato and spinach face a special pricing problem relative to retailers that sell non-perishable items. This is because before such grocers set prices for their produce, they have to take into consideration spoilage, cost as well as their rate of mark up. To take spoilage consideration, the grocer has to draw on his or her experience for the length of time it takes for a particular fruit or vegetable to rot. The following example shows the pricing procedure most grocers used.

Example 1

A grocer bought two boxes of cantaloupes (each box contains 200 cantaloupe) for $50. Overhead expenses related to the produce was $25. Based on experience, the grocer estimated that 25% of the 400 cantaloupes are expected to rot before being sold. The grocer wants 198% rate of profit (mark up) on cost. At what price must the cantaloupes be sold to achieve the desired mark-up?

Solution

Total profit = Mark up rate x cost

= 1.98 \times 50 = $99.00 \ Change \ the \ 198\% \ mark \ up \ to \ 1.98

selling price = mark up + cost + expenses

= $99.00 + $50 + $25

= $174

Expected quantities to sell = (100 – 25) \times 400 \ Calculate \ the \ number \ of \ cantaloupes

that will be sold.

= 0.75 \times 400 = 300 \ is \ the \ number \ that \ can \ be \ sold.

Selling price per cantaloupe = \frac{Selling price}{Quantity \ to \ sell} = \frac{174}{300} = $0.59
In the example, we used the number of items rather than the weight. The procedure we used in the above pricing process will not change, even if we were dealing with weights (in pound or kilogram). The following example illustrates this.

**Example 2**
A grocer bought 150 pounds of banana at $0.20/pound. The store wants a profit margin of 200% on cost. It estimates that 10% of the banana will rot before they are sold. How much should it sell one pound of the banana?

**Solution**

Total profit \( = (150) (0.20) \times 2.00 = $60.00 \)

Total selling price = cost + mark up
\[ = (150 \times 0.20) + 60 \]
\[ = 30 + 60 \]
\[ = $90.00 \]

Quantities expected to sell = \((100\% - 10\%) \times 150\)
\[ = 90\% \times 150 \text{ (then change 90\% to decimal)}\]
\[ = 0.90 \times 150 \]
\[ = 135 \text{ pounds will sell} \]

\[
\text{Selling price per pound} = \frac{90}{135} = $0.67 \text{ per pound}
\]

You might have noticed that the quantities that rot affect the unit-selling price. However, in practice, most grocers are compelled to cut down on their profit margin on perishable produce when they see that the produce are showing signs of rotting, or when they want to sell them quickly to prevent them from going bad. Experience is a valuable guide in predicting the perishability of a produce.
6.9 What are Basis Points?

A basis point (bps, for short) is a unit of measure used in finance to describe the smallest change in the value or rate of financial instrument such as stock, bonds and interest rate. One basis point is equal to 0.01% (or $\frac{1}{100}$th of a percent), or 0.001 in decimal form. If the Bank of Canada raises the interest rate by 25 basis points, it means the interest rate has been increased by 0.25% percent. If the existing interest rate is 4.20%, then the new rate of interest is 4.45% (4.20% + 0.25%). If it is expected that a stock index has moved up 124 basis points in the day’s trading, this means 1.25% increase in the value of the index. The table below summaries the difference between basis points, percent and decimal.

<table>
<thead>
<tr>
<th>Basis Points</th>
<th>Percent</th>
<th>Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.01%</td>
<td>0.0001</td>
</tr>
<tr>
<td>10</td>
<td>0.10%</td>
<td>0.0010</td>
</tr>
<tr>
<td>20</td>
<td>0.20%</td>
<td>0.0020</td>
</tr>
<tr>
<td>40</td>
<td>0.40%</td>
<td>0.0040</td>
</tr>
<tr>
<td>50</td>
<td>0.50%</td>
<td>0.0050</td>
</tr>
<tr>
<td>100</td>
<td>1.00%</td>
<td>0.010</td>
</tr>
<tr>
<td>1000</td>
<td>10.00%</td>
<td>0.100</td>
</tr>
<tr>
<td>10,000</td>
<td>100.00%</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>0.02%</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

From the above table, it may be obvious that to convert basis points to a percent divide by 100. Similarly, to convert percent to basis point simply multiply by 100. As well, to convert a percent to decimal form divide by 100.

6.10 Why Percent?

It is easier to compare percents in terms of their relative sizes than it is to compare fractions. Comparing percents is about comparing the numbers the percent sign is attached to, since the sign can be viewed as a common unit of measurement. For example, it is easier to compare 75% to 72%, than it is compare $\frac{3}{4}$ to $\frac{18}{25}$. This is why percent rather than fractions are used more often for comparison.
Math Skills for Business - Full Chapters

Percents are encountered often in our daily lives in a variety of situations: credit card interest rates, interest rate on OSAP, shopping (calculation of store sales discount), politics (to calculate support for various parties, issues and causes), business (to project increase or loss in volume of sales, comparing sales between two consecutive months), and in general (population growth, comparison among many items, rates of change in between periods). In most cases, percent is used to apportion cost or profit. Indeed, its use for sharing common costs or profit is more convenient compared to ratios or fractions.

6.11 Review, Exercises and Assignments

1. Express each of the following as percent.
   a) 10 out of 50   b) 5 out of 25   c) 3 out of 10   d) 260 out of 400
   e) 21 out of 70   f) 18 out of 60   g) 200 of 1000   h) 12 out of 48

2. The value of a one-bedroom condominium is $80000 and its contents are worth $3000. Express the value of the contents as a percentage of the value of the house.

3. A butter cake weighs 950 grams, of which 20% is sugar, 60% wheat flour, and 20% other ingredients.
   a) Calculate the weight of the sugar in the cake.
   b) What is the percentage difference between the amount of wheat flour and sugar in the cake?

4. In 2004, the total wages bill for a plastic manufacturing was $250000 and its total net sales were $2,500,000.
   a) Express the company’s wages bill as a percentage of the net sales.
   b) What interpretation will you give to the result in (a)?
   c) If the company’s wages bill expressed as a percent of its net sales in 2005 was 12%, state three factors that might have caused the increase.

5. Don bought a car with sale tax of 14% tax included for $31,920. In addition, Don bought an extended warranty at 2% of the original purchase price.
   a) What was the price of the car without the sale tax?
   b) How much was the extended warranty?
6. The value of a car decreases as shown in the table below:

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>New</td>
<td>$12,000</td>
</tr>
<tr>
<td>After 1 year</td>
<td>$10,000</td>
</tr>
<tr>
<td>After 2 years</td>
<td>$8,800</td>
</tr>
<tr>
<td>After 3 years</td>
<td>$8,000</td>
</tr>
</tbody>
</table>

a) In which year did the value of the car decrease by $2000?
b) Calculate the percentage decrease between year 2 and year 3.
c) During which year was the percentage decrease in the value of the car the greatest?

7. A gas bill of $46.06 includes GST of 6%. Find the amount of the GST paid.

8. The year-end profit of a company increased this year by 12% to $90,944. What was the profit made last year?

9. In a massive sale event, the following items were offered at discount prices as listed:

<table>
<thead>
<tr>
<th>Item</th>
<th>Sale Price</th>
<th>Discount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Television</td>
<td>$299.00</td>
<td>10%</td>
</tr>
<tr>
<td>VCR</td>
<td>$273</td>
<td>12%</td>
</tr>
<tr>
<td>Computer</td>
<td>$2,200</td>
<td>25%</td>
</tr>
<tr>
<td>Calculator</td>
<td>$24.99</td>
<td>5%</td>
</tr>
<tr>
<td>Colour Printer</td>
<td>$225</td>
<td>10%</td>
</tr>
</tbody>
</table>

What were the prices of these items before the sale?

10. After one year, the value of a car has fallen by 15% to $8,330. What was the value of the car at the beginning of the year?

11. A real estate agent receives as commission of 6% of the sales price of the property she sells. If a house is sold for $380,000, what is the agent’s commission? How much does the seller of the house receive?
12. Musty Pharmacy marks up its prescription drugs by 42%. If a certain drug costs $13 from the wholesale distributor, what will be the selling price at the Musty pharmacy?

13. AB-Mart plans August 25 sale in which they will mark down garden tools by 15%. What will be the sale price of a wheelbarrow that originally costs $60?

14. The selling price of a suit of clothes is $200. If it costs $150 at factory prices, what is the percent of mark-up based on the selling price?

15. George was hired at a yearly salary of $38,000. He received a raise of 11% and then a personal merit-raise of 6%. What is his current salary after the two raises?

16. A cotton picking machine picks up 80% of the cotton each time it goes over the field. If a farmer runs his machine over a field twice, what percentage of the cotton will be harvested?

17. At a tire factory, 0.5% of the tires are rejected because of various defects. The factory manufactures 1800 tires per day.
   a) How many tires are rejected each day?
   b) How many tires are accepted each day?

18. A store buys ladies’ blouse for $25.50 and sells them to its customers for $62.00. What is the percent mark-up of the blouse based on cost?

19. Bobby bought a stereo system for $450, which was 70% of the regular price. How much did the stereo cost originally?

20. A basketball net was originally priced at $205.00. It has been marked down twice: 10% off the original price, then another 15% off the discounted price.
   (a) What is the new price for the net?
   (b) What is the single discount rate?

21. Paul bought a new pair of shoes. The sales tax of his purchase was $4.80. The sales tax is 14%. How much did he pay for the shoes?

22. A pair of rollerblades is selling for $140. Fortunately, they are on sale today at 20% off.
   a) What is the sale price of the blades?
   b) How much will one pay, in total, for the blades after 14% sales tax is added?
23. Sharika and Mike’s restaurant bill came to $42.25.
   a) Calculate the tax (which is 11%).
   b) Calculate the tip (which is 15% of the total after tax).
   c) How much will Sherika and Mike pay for their meal?

24. Steve bought a small one-bedroom condominium for $76,000 and a used car for $20,000. He sold the condo at a gain of 15% and the car at a loss of 12%. Find the total amount gained or lost from the two transactions.

25. Kim bought a painting antique for $140. Two years later, its value increased by 25%. What was the new value of the painting?

26. Calculate the percentage increase in each of the following cases.
   a) A transit fare of $1 is now $1.20.
   b) A taxi fare of $2.20/km is now 2.35/km.
   c) A train fare of $3.15 is now $3.50.

27. A used car dealer bought a used car and spent $800 on repairs. He sold the car for $18000, gaining 20% on the purchase price. How much did he buy the car?

28. In a year, the value of a town house increased from $460000 to $480000. Find the percentage increase in the value of the house and use it to estimate the value after another year.

29. A clothing store offers $10 discount to all customers spending $50 or more. Karen spends $52.63, and Mike spends $78.82. Find the percentage saving for Karen and Mike.

30. Ann, an investment broker, bought some shares at a price of $5.50 each. The price of the shares dropped on the stock market to $3.50. Find the percentage loss on 200 shares.

31. Janet wants to buy a used car that cost $4000. The salesperson requires \( \frac{2}{5} \) of the price as deposit. Janet’s mother gives her 20% of the price. Do you think this is enough? Explain your answer.

32. Calculate the equivalent single rate of discount for each of the following series of discounts. a) \( \left(18 \frac{2}{3}\% \right) \text{ then } 8.5\% \)  b) 25% then 8 \( \frac{1}{2}\% \) followed by 2%

33. A retailer lists an item for $95 less 20% discount. To improve turnover, the item is reduced again to $60. What additional rate of discount should be offered?
34. A double-door refrigerator listed at $1136.00 has a net price of $800. What is the rate of discount?

35. Compute the equivalent single rate of discount for each of the following series of discount. a) 30%, 12.5%  
   b) 25%, 20%, 30%.

36. A computer listed at $975.00 is sold for $820. What is the rate of discount that was allowed?

37. Joe’s salary for the next year is to be $49,700. This represents 8% increase over this year’s salary. What is Joe’s present salary?

38. A retailer has some sweaters that cost her $38 each. She wants to sell them at a profit of 20% of her cost. What price should she charge for the sweaters?

39. John buys calculators for $5 each and sells them to other students for $7.50. Find his percentage profit based on cost.

40. Convert each of the following percentages to fractions, giving your answers in the simplest form.
   a) 10%  
   b) 80%  
   c) 90%  
   d) 5%  
   e) 25%  
   f) 75%  
   g) 35%  
   h) 38%  
   i) 24%  
   j) 12%  
   k) 72%  
   l) 50%  
   m) 14%  
   n) 20%  
   o) 18%  
   p) 40%

41. Mercy earned $60,000 in 2001. Her tax allowance was $6490. She did not pay tax on this amount of her income. On further $2,500 of her income, she did not pay any income tax because she paid this amount into a pension scheme. She, however, paid tax on the rest of her income. The tax rates are as follows: 25% on the first $30,000 of her taxable income; then 40% on the remaining.

   a) What was her taxable income? (Note: Taxable income is what remains after all allowances are subtracted.)

   b) How much income tax did Mercy pay in 2001?
42. a) Find the taxable income of a family of 6 (husband, wife, and four children) whose adjusted gross income (before deductions) is $52,000 and itemized allowances and deductions are $10,873.
   b) What percentage of the family’s income are allowances and deductions?

43. Every year a municipal council determines its budget and decides how much it will tax properties under its area. The process of determining the value of a home is called assessment. A council may use either the historic cost of the property (its cost price) or market cost (How much the property will sell in the market at the time of assessment).
   a) Find the assessed value of a store with a market value of $150,000 if the rate of assessed value is 35% of market value.
   b) What is the tax on a property with assessed value of $188,500 if the tax rate is 4.5% of the assessed value?
   c) Jack’s farm has a market value of $400,000. Find the assessed value of the farm if farms are assessed at 25% of the market value.

44. A stereo sells for $145, which includes a mark-up 65% of its cost. Find the cost and the mark up.

45. A cookbook has a 34% mark-up based on cost. If the mark –up is $5.27, find the cost of the cookbooks.

46. A file cabinet originally sold for $129 had scratches on it and had to be reduced to 125. What was the rate of discount or markdown based on the original selling price?

47. A sports stadium has a seating capacity of 53,983. If 47,892 fans attended a game, what percent of the seats were filled?
48. M&E Grocery Inc  
Income Statement for the Year Ended July 31, 2006

<table>
<thead>
<tr>
<th>Amount</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Sales</td>
<td>$250,000</td>
</tr>
<tr>
<td>Cost of Sales</td>
<td>180,000</td>
</tr>
<tr>
<td>Gross margin</td>
<td>70,000</td>
</tr>
<tr>
<td>Operating expenses</td>
<td>37,000</td>
</tr>
<tr>
<td>Net Income</td>
<td>$33,000</td>
</tr>
</tbody>
</table>

a) Complete the percent column by calculating each amount as a percent of the net sales.  
b) State two ways the grocer could increase its net income.

49. A 5 1/4-inch double-sided, double-density floppy disk costs $0.99 and sells for $1.50.  
a) Find the rate of mark-up based on cost.  
b) Find the rate of mark-up based on the selling price.

50. A refrigerator that sells for $389.99 was marked down $97.00. What is the sale price?

51. Explain why taking a series discount of 25% and 30% is not the same as taking a single discount of 55%. Explain your answer with a specific example.

52. A camcorder that originally sold for $900 was reduced to sell for $799. It was then reduced additional 40%. Calculate a single equivalent discount.

53. The telephone costs of a company last year were $10,000, including sales tax of 12.5%. It was decided to allocate 60% of these phone costs, excluding the sales tax, to central Administration, and to allocate 30% of the reminder, excluding sales tax to finance. How much was allocated to finance (To the nearest dollar)?

54. In a quality check of a batch of components, \( 1 \frac{1}{2} \% \) are rejected. If a total of 39 components are rejected, calculate the number in the batch.
55. Last week a car dealership sold 12 cars. A new sales promotion came out, and, as a result, the next week they sold 19 cars. What was her percent increase in the sales?

56. A car dealer sells SUV for 39,000%, which represents a 25% profit over cost. What was the cost of the SUV to the dealer?

57. What were your total sales if
   a) you made $750 at a commission rate of 15%
   b) you collected a 22% fee that amounted to $16,500?

58. Vida works strictly on a commission basis by selling computer hardware. She receives a 20% commission on all sales of computer hardware she closes. Vida’s goal for a gross salary is $60,000. How much computer hardware must Vida sell in order to meet her target salary?

59. Seventy percent of a town’s population voted in an election. If 1,589 people voted, what is the population of the town?

60. The finance officer for an accounting firm allows $3,400 for supplies in the annual budget. After three months, $898.32 has been spent on supplies. Is this figure with 25% of the annual budget? Explain your answer.

61. A math student answered 60 questions correctly on a 100-question test. What percent of the questions were answered correctly. What percent of the questions were answered correctly?

62. Pete is the accountant of Family Grocer. He calculates the selling price for all potatoes. If 500 pounds of potatoes were purchased for $0.20 per pound, and 18% of the potatoes were expected to rot before being sold, calculate the price per pound that the potatoes must sell for if a profit of 120% of the purchase price is desired.

63. A wallet costs $12.05 to produce. The wallet sells for $21.68. What is the rate of mark up based on cost?

64. A bookcase desk that originally sold for $129.99 was marked down 25%. During the sale it was damaged and had to be reduced again by 50%. What was the final selling price of the desk?
65. Laura purchased a small table top refrigerator for her room for $95.99. The price included a mark up of $27.20 based on the cost.
   a) Find the manufacturing cost of the fridge.
   b) What is the percent rate of mark up based on cost?

66. Loose-leaf paper in a university book store is marked 30% of its cost. What is the cost of the loose-leaf paper if the selling price is $2.34 per package?

67. Last month a car dealership sold 12 cars. A new sales promotion came out the first week of the following month and she sold 19 cars. What was the percent increase in sales from last month compared to following month?

68. Jenny sold 80% of the tie-dyed shirts she took to the Caribbean festival. If she sold 45 shirts, how many shirts did she take?

69. Ms. Lu purchased a women’s magazine at the Halifax airport for $2.99. The tax on the purchase was $0.36. What is the tax rate at the Halifax airport?

70. A stockbroker sold her shares and made a profit of $1,566. If this a profit of 23%, how much were the shares worth when she originally purchased them?

71. Dora purchased miscellaneous items at Zellers. The item subtotaled $52.13. The receipt showed $4.17 in tax paid. What is the tax rate?

72. At John’s Wear store, all suits are reduced 20% from their retail price. Charles purchased a suit that originally retailed for $258.30. How much did he save?

73. In a quality check of a batch of components, $1 \frac{1}{2}$% were rejected. If a $39$ components were rejected, calculate the number in the batch.

74. Last year a software company sold 6,600 products in a particular price range. The target for the current year is 11,500. What percent increase does this represents?

75. A customer left 15% tip with a restaurant receipt. The receipt totaled $29.45 including the tip, find the amount of the tip. What was the total cost of the meal, including the tip?
76. In an internal audit of 200 sale invoices, the following errors were discovered:

<table>
<thead>
<tr>
<th>Number of errors</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of invoices</td>
<td>60</td>
<td>30</td>
<td>40</td>
<td>40</td>
<td>20</td>
<td>10</td>
</tr>
</tbody>
</table>

What is the percentage of invoices with errors?

77. A new lake is to be stocked with fish according to the numbers in the table below.

<table>
<thead>
<tr>
<th>Type of fish</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of fish</td>
<td>400</td>
<td>300</td>
<td>200</td>
<td>100</td>
</tr>
</tbody>
</table>

What is the percentage of fish type D would be in the lake?

78. Six employees at a meat packing plant were sick on Monday. If the plant employs 480 workers, what percentage of the employees were sick on Monday?

79. 30% of 27 equals what number? Is it greater than 20 percent of 30? Explain with illustration.

80. Allison takes $1,400 monthly retirement income. In January, she received a % cost of living allowance. How much was the increase?

81. Explain one instance when you have used percent to solve a problem (outside of this course or homework).

82. Twenty-six percent of all forest fires are caused by lightning. There are about 2,500 forest fires in Canada. About many of these fires are caused by lightning?

83. Kelly Cleaners raised the price of dry cleaning a jacket from $5 to $6. The same percentage increase was applied to the price of dry cleaning a coat. What is the present cost of cleaning a coat if the previous cost of cleaning a coat was $10.00?

84. When 4131 people attended a concert, the concert hall was 80% full. What is the capacity of the hall?

85. The price of a major newspaper rises from 55 cents to 75 cents. What is the percent increase in price?

86. A grocer bought 10 crates of oranges for a total cost of $80. If it lost 2 crates due to spoilage, at what price would it have to sell each the remaining crates in order to earn a total profit of 25% on cost?
87. Three people shared a taxi to Toronto airport from Brampton. The fare was $36.00 and they gave the driver a tip equal to 25% of the fare. If they shared the cost equally, including the tip, how much did each person pay?

88. A store sells a watch for a profit of 25% of the wholesale cost. What percent of the selling price of the watch is the store’s profit?

89. After getting a 20% discount, Eddy paid $100 for a bicycle. How much, in dollar did the bicycle originally cost?

90. What is the similarity between percent and ratio?

91. Ian went to a fashion store to buy a leather jacket. He found one which regularly sells for $145 but was on sale for 20% off. The cashier takes the regular price of $145, adds on the 14% sales tax and then applies the 20% discount to the whole amount. However, Ian objects that he was being overcharged for the jacket. He tells that cashier that she should apply the discount first to the $145, and then apply the sales tax to the discounted price. Do you think Ian is right?

92. The interest rate is 5.25%. To control excessive consumer spending, the Bank of Canada decided to raise the interest rate by 2 basis points. What is the interest rate now?

93. In his will, Mr. Kitson left 25% of his estate to his wife and unevenly divided the balance between his son and daughter. If the son received $36,000 as his share, what was the total value of his estate?

94. Sean uses a discount voucher to reduce the price of petrol by cents per litre. The original price is 101.83 per litre. By what percent is the price reduced?

95. The table below shows the consumption of water per person during certain years.

<table>
<thead>
<tr>
<th>Year</th>
<th>1970</th>
<th>1976</th>
<th>2004</th>
<th>2021</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>120</td>
<td>160</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) A 275% increase in consumption is predicted from 2004 to 2021. Calculate the predicted consumption in 2021.
b) Calculate the percent increase in consumption from 1976 to 2004.
c) Consumption in 1976 was 20% more than the consumption in 1970. Calculate the consumption in 1970.

96. a) Convert 15 basis points to percent.
   b) Convert 0.060 to basis points.
97. The cost of a product is broken down as follows;
   Material cost…….. $20
   Labour cost ……. 40
   Variable overhead…..10
   Fixed overhead…… 15
   What percent is the labour cost?

98. Material cost …….$25
   Labour cost ………33
   Variable overhead 12
   Total Variable cost $70
   Therefore, contribution margin = selling price – total variable cost
   \[ \text{contribution margin} = \text{selling price} - \text{total variable cost} \]
   \[ = $140 - $70 = $70 \]
   What is the percent of contribution margin to selling price?

99. The student enrolment at Zimp Community College increased from 900 to 1,200.
   Find the percent increase to the nearest whole percent.

100. Mel’s monthly salary as a teacher increased from $2,500 to $2,720. Find the percent increase.

Case Study # 1
Megton is a local athletic shoe manufacturer that makes running shoes at a cost of $8.40 a pair. An inspection report indicates that 10% of the running shoes will be defective and must be sold to Odd Tops Inc. as irregulars for $12 per pair. Megton produces 2,000 pairs a month.

a) If it desires a mark-up of 100% on cost, find the selling price per pair of the regular running shoes.

b) Calculate Megton’s gross revenue for a month.

c) Megton has implemented a quality improvement initiative that has reduced the percentage of defective running shoes by 6%. Recalculate the selling price per pair based on this new information.
Case Study #2

Extra Corner Grocery Company purchased 200 pineapples for $180.00. It is expected that about 20% of the pineapples will rot after one month before they are sold. And another 20% will rot after two months before being sold. The store desires a mark up of 100% on cost.

a) The store was able to sell 150 pineapples within one month. At what price did it sell the pineapples to achieve the desired profit margin?

b) The other 50 pineapples were sold after one month. At what price did it sell them to achieve the desired profit of 100% mark up?

c) Assume that the store had to reduce the price of each of the 50 pineapples by 15% in order to sell them quickly. What was the selling price of each? How much was the store’s mark up?

Case Study #3

Jessalyn has been saving to buy some camping equipment from a store. The table below shows the prices of the equipment she needs.

<table>
<thead>
<tr>
<th>Item</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tent</td>
<td>$250</td>
</tr>
<tr>
<td>Sleeping bag</td>
<td>$100</td>
</tr>
<tr>
<td>Boots</td>
<td>$55</td>
</tr>
<tr>
<td>Cooking kit</td>
<td>$35</td>
</tr>
</tbody>
</table>

a) The camping store is offering a 20% discount this weekend on each of the camping items listed in the table. After the 20% discount and before tax is added, what is the price of the tent?

b) Jessalyn must pay a sales tax of 14% for her total purchase. Including the discount and after the tax is added, how much will it cost Jessalyn to buy all of the four pieces of equipment in the table?

c) Jessalyn has a coupon that will allow her additional 10% off the discounted price of the sleeping bag. If she uses the coupon to buy the sleeping bag, what is the final price of the sleeping bag after tax is added?
Case Study # 4

Businesses that rent their premises in shopping centres or malls usually are required by their leases to pay a certain percent of the common maintenance cost of the shopping centres or malls. These common maintenance expenses include parking lot maintenance, garbage collection, municipal taxes, sign maintenance, security services, general customer services and other expenses which are part of running a shopping centre or mall. The amount each business pays for these common maintenance costs depend on the space it occupies in relation to the total space of the shopping centre or mall. That is, each percent of the common maintenance costs is based on the space footage per store and the total space footage of the whole shopping centre or mall.

For example, a store that occupies 6,500 square feet in a shopping mall of 72,000 square feet, will be required to pay the following percent of the common maintenance costs:

\[
\frac{6,500}{72,000} \times \frac{100}{1} = 9.03\%
\]

If the common maintenance cost is $7,800.25, that store’s share of the cost = $7,800.25 \times 0.0903 = $704.36.

a) (i) Maxima Fashion occupies 1,500 square feet in Rock Shopping Mall, that is 8,500 square feet. What percent of the mall’s space does it occupy?
(ii) If the total common maintenance cost of the mall is $16,900.17, excluding taxes, how much should Maxima Fashion pay?

b) A lease requires that Super Computer Services, occupying 1,800 square feet of a mall of 79,500 square feet, to pay a certain percent of the yearly taxes based on space footage occupied.
(i) If the taxes for the summer months are $28,000, how much must Super Computer Services pay?
ii) During the winter season, the common maintenance cost of the mall goes up due to snow removal, deicing, heating and frequent cleaning. If the common maintenance cost for the mall for the winter period is $79,714.83, how much should Super Computer pay?

c) Delight Treat occupies 1000 square feet of a 58,800 square-foot shopping Centre called Shopping Village. The total maintenance cost is $67,900. Delight Treat’s owner compares that cost to two other malls of approximate square footage and finds Shopping Village charges too much. After a series of meetings with the mall management, it was decided to cut security services by $2,500 and general customer services by $10,000. How much should Delight Treat pay for the common maintenance cost after the cost cutting?

d) Modern Sports occupies 900 square feet in the same shopping mall as Super Computer Services. The owner contends that the common maintenance cost should not be based on square footage, because that gives an unfair advantage to certain stores. The owner further contends that occupant common maintenance cost should be based on two key factors instead of one: space occupied and the volume of sales. Do you agree with the owner of Modern Sports? Explain your response.
A graph may be regarded as a pictorial representation of data. A graph may be drawn manually on paper, or by using software on a graphing calculator or computer. Graphs, especially of functions, are mostly drawn on a rectangular co-ordinate plane. A rectangular co-ordinate plane is formed by the intersection of two number lines. One, on the East-West directions is called the horizontal axis, and the other on the North-South directions is called the vertical axis. The horizontal and vertical axis make up the co-ordinate axis, and divide the co-ordinate plane into four regions called quadrants (fig. 1).

Why a Co-ordinate Plane? Because a point on a co-ordinate plane is also identified as the ordered pair of numbers (horizontal co-ordinate, vertical co-ordinate), and an ordered pair of numbers interpreted as (horizontal co-ordinate, vertical co-ordinate) is identified as a point on a co-ordinate plane. The horizontal co-ordinate is the number on the horizontal axis, and the vertical co-ordinate is the number on the vertical axis associated with the point on the co-ordinate plane.
In (fig. 2) the points A, B, C, D are also identified by the attached pair of numbers. The ordered pairs (2, 3), (-5, 4), (-3.5, -6), (3, -2.5) are the points attached.

**NOTE:** In fig. 2, the letter x is representing the horizontal axis and in such a case the horizontal axis becomes the x-axis, and similarly the vertical axis is the y-axis. It should be noted that the axis can be assigned any names.

Graph of Functions:

What is a function? For our purpose, it is a set of instructions and procedures which takes a number called the INPUT NUMBER to produce another number called the
OUTPUT NUMBER. The set of instructions and procedures are given either by words (e.g. XX), or by algebraic equations (e.g. YY). Therefore the function is also given by the set of ordered pair of numbers (INPUT NUMBER, Corresponding OUTPUT NUMBER) of the function for all possible input numbers.

Function is:

And also the set of ordered pair of numbers (INPUT NUMBER, OUTPUT NUMBER) for each input number and its corresponding output number.

The ordered pair of numbers (INPUT NUMBER, OUTPUT NUMBER) is also a point on a co-ordinate plane whose horizontal axis is the INPUT NUMBERS and the vertical axis is the OUTPUT NUMBERS. For such a coordinate plane the horizontal axis is called the INPUT-axis, and the vertical axis the OUTPUT-axis.

The graph of a function is always on such a coordinate plane, and it is all the points with coordinates (INPUT NUMBER, corresponding OUTPUT NUMBER).

How is the graph of a function constructed?

1. Choose a letter (e.g. t) or word (e.g. distance) to represent the INPUT numbers, and a different letter (e.g. q) or (e.g. amount) to represent the OUTPUT numbers if they are not given. From the examples, the horizontal axis is then the t-axis and the vertical axis the q-axis.

2. Make a table (of values) with the headings INPUT (eg t), OUTPUT (eg q), and (INPUT, OUTPUT) {eg (t, q)}. From the possible INPUT numbers choose about 11 different numbers if the function if not linear and 4 numbers if it is a linear function. For each INPUT number chosen find the corresponding OUTPUT number and the ordered pair of numbers (INPUT, OUTPUT).

3. With the INPUT and OUTPUT numbers from (2) as a guide choose a scale for the INPUT (horizontal) and OUTPUT (vertical) axis and draw both axis.
4. Plot the points for the ordered pair of numbers found in (2). If all the points are on a straight line join them by a straight edge (e.g. ruler) and extend in both directions. Otherwise join the points by a curve in order of magnitude of the INPUT numbers, and extend in both directions.

5. Give a title which identifies the function or the application to your graph. All the functions in the following examples are linear and so the graphs are straight lines. By Euclid we need only two points, but we add an extra or two as a check.

Example 1

Dora works a night shift in a warehouse and earns $10/hour plus $2.00 premium bonus per hour. Draw a graph for Dora’s earnings given the amount of time worked. Use your graph to estimate how much she will earn for working 8 hours.

Solution

The INPUT is the amount of time worked and OUTPUT is amount earned, since earning is for work done. Amount of time worked in hours is represented by t and the amount earned (wages) in dollars by w. The horizontal axis is therefore the t-axis, and the vertical axis the w-axis. The graph may be titled “The Wages of Dora”.

From the question Dora’s hourly earnings = $10 + $2 = $12. On the assumption that Dora’s hourly earnings is unchanged, with hours worked we make the calculations:

<table>
<thead>
<tr>
<th>t in h</th>
<th>w in $</th>
<th>(t,w)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>(1,2)</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td>(2,24)</td>
</tr>
<tr>
<td>4</td>
<td>48</td>
<td>(4,48)</td>
</tr>
<tr>
<td>5</td>
<td>60</td>
<td>(5,60)</td>
</tr>
</tbody>
</table>
Example 2:
A computer technician charges $30 for diagnosing computer problems and $20/hour for fixing the problems. How much will she charge for work done on a computer which took her $2\frac{1}{2}$ hours after diagnosis to fix? (Find solution by graph).

Solution:
The INPUT number is the amount of time in hours, $t$, after diagnosis it takes technician to fix problem. The OUTPUT number is the amount in dollars, $a$, the technician charges for diagnosis and fixing the problem. The horizontal axis is therefore the $t$-axis, and the vertical axis the $a$-axis. The graph may be titled “Fee for Diagnosis and Repair”.

On the assumption that the diagnosis fee and the hourly rate is unchanged by the amount of time it takes to diagnose and/or fix, we make the following calculations:

| $t$ in h | $a$ in $|$ | $(t, a)$ |
|---------|-----------|---------|
| 0       | $30 + 20(0) = 30$ | $(0, 30)$ |
| 1       | $30 + 20(1) = 50$ | $(1, 50)$ |
| 2       | $30 + 20(2) = 70$ | $(2, 70)$ |
| 3       | $30 + 20(3) = 90$ | $(3, 90)$ |
From the two arrows on the graph, she will charge $80 for Diagnosis and 2\frac{1}{2} \text{ hours repair time.}

*Exercise:* Show that \( a = 30 + 20t \)

**Example 3**

The cost in dollars ‘C’ of preparing meals for ‘m’ number of people is given by the equation:

\[ C = 8m + 25 \]

a) Draw a graph of the equation.

b) From the graph, what is the cost of preparing meals for 9 people?

c) From the graph, how many people can be fed by $121?

**Solution:**

‘C’ is expressed in terms of ‘m’, and by convention the m-values are the INPUT and the C-values are the OUTPUT numbers. Therefore the horizontal axis is the m-axis, and the vertical axis is the C-axis. ‘m’, is the number of people, so a value of m cannot be negative. Within this constraint choose any four numbers for m, to make the Table of Values. The title of the graph is the equation: \( C = 8m + 25 \).
a) Graph the relationship between unit price and the quantities demanded per week.

b) Use the graph to find out the demanded quantities when the price fell to $10.00.

c) How much will be the gross income of the toy producer when 30,000 were demanded at $20.00 a toy?

**Solution**

The p-values are the INPUT numbers, and the q-values are the OUTPUT numbers, therefore the horizontal axis is the p-axis and the vertical axis is the q-axis. It is
assumed that the graph of demand per week (q) against unit price is a straight line. From the table in the question the table of values of the graph is made:

\[
\begin{array}{|c|c|c|}
\hline
p & q & (p, q) \\
\hline
20 & 30,000 & (20, 30,000) \\
25 & 20,000 & (25, 20,000) \\
30 & 10,000 & (30, 10,000) \\
\hline
\end{array}
\]

b) From the arrows on the graph, at $10 per unit price the demand is 50,000 toys.
c) Gross Income in $ = 20 (30,000) = 600,000. Gross Income is $600,000.

Comments on:
Example 1: (i) The points (3, 36) and (7, 84) are on the graph. \(\text{(Please check)}\)
The difference of the vertical coordinates of these two points = 84 – 36 = 48.
The difference of the corresponding horizontal coordinates = 7 – 3 = 4.

\[
\frac{\text{The difference of the vertical coordinates of the two points}}{\text{The difference of the corresponding horizontal coordinates}} = \frac{48}{4} = 12
\]

For any other two points on the graph, similar calculation will give the same result 12. \(\text{(Pick any two points on the graph, to check the claim.)}\)

(ii) The mathematical relationship between the vertical coordinates w, and the horizontal coordinate t of points on the graph is given by the equation: \(w = 12t\).

Example 2: (i) The points (1.5, 60) and (4, 110) are on the graph. \(\text{(Please check)}\)
For any other two points on the graph, similar calculation will give the same result 20. *(Pick any two points on the graph, to check the claim.)*

(ii) The mathematical relationship between the vertical coordinates, \(a\), and the horizontal coordinates \(t\) of points on the graph is given by the equation: \(a = 20t + 30\).

Example 3: The graph is for the equation: \(C = 8n + 25\). Pick any two points on the graph and check that: 

\[
\frac{\text{The difference of the vertical coordinates of the two points}}{\text{The difference of the corresponding horizontal coordinates}} = 8
\]

Example 4: (i) The points (13, 4400) and (23, 24000) are on the graph. *(Please check)*

\[
\frac{24000 - 44000}{23 - 13} = \frac{-20000}{10} = -2000
\]

For any other two points on the graph, similar calculation will give the same result -2000. *(Pick any two points on the graph, to check the claim.)*

(ii) The mathematical relationship between the vertical coordinates \(q\), and the horizontal coordinate \(p\) is given by the equation: \(q = -2000p + 70\,000\).

### 11.2 Linear Relations

A linear relation between an OUTPUT variable (e.g. \(y\)) and an INPUT variable (e.g. \(x\)) is geometrically a straight line graph, and mathematically an equation usually of form \(y = mx + c\). So examples 1, 2, 3, 4 are linear relations, and \(y\), \(x\), \(m\), and \(c\) are:

<table>
<thead>
<tr>
<th>Examples</th>
<th>(y)</th>
<th>(x)</th>
<th>(m)</th>
<th>(c)</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(w)</td>
<td>(t)</td>
<td>12</td>
<td>0</td>
<td>(w = 12t).</td>
</tr>
<tr>
<td>2</td>
<td>(a)</td>
<td>(t)</td>
<td>20</td>
<td>30</td>
<td>(a = 20t + 30)</td>
</tr>
<tr>
<td>3</td>
<td>(C)</td>
<td>(n)</td>
<td>8</td>
<td>25</td>
<td>(C = 8n + 25) (given)</td>
</tr>
<tr>
<td>4</td>
<td>(q)</td>
<td>(p)</td>
<td>-2000</td>
<td>70,000</td>
<td>(q = -2000p + 70,000)</td>
</tr>
</tbody>
</table>
The ‘m’ is called the slope or the gradient of the line graph (in analogy with an incline). It is also referred to as the rate of change of the OUTPUT value to a unit change of the INPUT value (in analogy to applications as in the examples). \((0, b)\) is called the y-intercept, and is the point of intersection of the line graph and the vertical axis. These two values the slope \(m\) and the point \((0, b)\) describe both the graph and its equation. They can be used to draw the graph and/or write the equation of the graph in the **slope intercept form**: \(y = mx + b\).

**Interpreting Linear Graphs:**

*If the graph trends upwards from left to right, it indicates that as \(x\) increases \(y\) increases, and as \(x\) decreases \(y\) decreases and the Slope is positive.* (eg 1, 2, 3).

A classic example is Price and Demand ‘curve’.

*If the graph trends downwards from left to right, it indicates that as \(x\) increases \(y\) decreases, and as \(x\) decreases \(y\) increases and the Slope is positive.* (eg 4).

A classic example is Price and Supply ‘curve’.

*If the graph is parallel to the \(x\)-axis (horizontal line), then an increase or decrease in \(x\) causes no change in \(y\) that is \(y\) has a fixed value, and the Slope is zero.*

*If the graph is parallel to the \(y\)-axis (vertical line), then \(x\) has a fixed value that causes \(y\) to take all values, and the Slope is undefined.*

**The Slope, \(m\), given the line:**

The slope of a line is found from any two points on it (given or taken) e.g. \((x_1, y_1)\) and \((x_2, y_2)\) and is:

\[
m = \frac{\text{The difference of the vertical coordinates (y) of the two points}}{\text{The difference of the corresponding horizontal coordinates (x)}} = \frac{y_2 - y_1}{x_2 - x_1} \quad \left(\text{or} \quad \frac{y_1 - y_2}{x_1 - x_2}\right)
\]

It is important to note that: \(x_1 \neq x_2\) (\(x_1\) cannot be equal to \(x_2\)).

This is also the first step in finding the equation of a line or line through given points.
Example 5

The table below shows the hourly earnings (in dollars) for daycare workers in a wood manufacturing company in Ontario from 2001 to 2005.

<table>
<thead>
<tr>
<th>Calendar year x</th>
<th>2001</th>
<th>2002</th>
<th>2004</th>
<th>2005</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hourly rate y in $</td>
<td>14.20</td>
<td>14.50</td>
<td>15.10</td>
<td>15.40</td>
</tr>
</tbody>
</table>

Find the rate of change in the hourly rate per year of the workers from 2001 to 2005.

Solution:

Plot the points. If they lie on the same straight line then the rate of change for any two periods of time is the same, and is equal to the slope of the line.

(2001, 14.20) and (2002, 14.50) are on the line.

\[ \text{slope } m = \frac{14.50 - 14.20}{2002 - 2001} = \frac{0.30}{1} = 0.3 \]

Therefore the rate of change in hourly earnings between 2001 and 2005 is $0.30 / year.

Example 6

Find the slope of the line containing (-1, 0) and (3, 4).

Solution

It is given that \( P_1 = (-1, 0) \) and \( P_2 = (3, 4) \) are on the line.

\[ \text{Slope } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{3 - (-1)} = \frac{4}{3 + 1} = \frac{4}{4} = 1 \]

Slope of the line, \( m = 1 \).
Example 7

Show that if a graph is a horizontal line, then its slope is 0.

Solution

Horizontal line is parallel to the x-axis. Any point on the line is of form \((h, k)\), \(h\) is any number, \(k\) is a fixed number and \((0, k)\) is the y-intercept.

The two points \((h_1, k)\) and \((h_2, k)\) are on the horizontal line, and so the slope of the line is

\[
m = \frac{k - k}{h_2 - h_1} = \frac{0}{h_2 - h_1} = 0 \quad h_2 \neq h_1
\]

Example (-2, 5) and (3, 5) are on the graph shown,

Therefore the slope \(m = \frac{5 - 5}{3 - (-2)} = \frac{0}{5} = 0\)

Example 8

Show that if a graph is a vertical line, then its slope is undefined.

Solution

Vertical line is parallel to the y-axis. A point on it is of form \((k, v)\), \(k\) fixed, and \(v\) is any number.

The two points \((k, v_1)\) and \((k, v_2)\) are on such a vertical line, and so the slope of the line is

\[
m = \frac{v_2 - v_1}{k - k} = \frac{v_2 - v_1}{0} = \text{undefined} \quad v_2 \neq v_1
\]

Example (2, 5) and (2, 2) are on the graph shown,

Therefore the slope \(m = \frac{5 - 2}{2 - 2} = \frac{3}{0} = \text{undefined}\)
11.3 Graph of the linear equation: 

$$ax + by + c = 0$$

In general, a linear equation of $x$ and $y$ is of form: $Ax + By + C = 0$.

We can also solve this to give: $y = mx + b$

$$m = \frac{-A}{B}, \quad c = \frac{-C}{B}.$$  

$m$ is the slope, $(0, b)$ the $y$-intercept, $(\frac{-C}{A}, 0)$ the $x$-intercept of the graph of the equation $Ax + By + C = 0$.

The equation of form $y = mx + c$ is called the slope-intercept form (for obvious reasons).

*Take Note:* From the equation the slope, the $y$-intercept, and the $x$-intercept of the graph can be determined before it is drawn. From this information it can be deduced whether the graph is horizontal (if $A = 0$), vertical (if $B = 0$), trends upwards (if $m$ is positive) or downwards (if $m$ is negative) from left to right. The information can also be used to draw the graph of the equation.

**Solutions of equation and points on its graph**

The pair of numbers $x = h, \quad y = k$ or $(h, k)$ is a solution of the linear equation if: $ah + bk + d = 0$

1. A solution $(h, k)$ of an equation is the coordinates of a point on its graph.
2. The coordinates of any point on a graph, is a solution of its equation.

11.4 Drawing Graph of Linear Equation

The graph of a linear equation is a straight line. By Euclid a straight line is defined by any two points on it because there is only one straight line that passes through two points. Combined with the first statement above, any two solutions of the linear equation can be used to draw its graph. That is, plot the two points and draw a straight line extended at both ends to pass through the two points.

**Methods of Solution of Linear Equation:**

i. Guess and verify. For most equations it is inefficient and impossible for some.

ii. Substitute any number for $x$ in the equation and solve to find corresponding $y$.

iii. Substitute any number for $y$ in the equation and solve to find corresponding $x$. 

**y-intercept:** Is the point of intersection of the graph and the y-axis. A point on the y-axis is of form (0, k). So to find the y-intercept, 0 is substituted for x in method (ii).

**x-intercept:** Is the point of intersection of the graph and the x-axis. A point on the x-axis is of form (h, 0). So to find this point, 0 is substituted for y in method (iii).

*It is often most efficient to combine methods (ii) and (iii) and/or the intercepts.*

**Example 9: (Find and use intercept to draw graph).**

a) Find the x- and y-intercepts of the line 4x - 2y = 8

b) Graph the equation.

**Solution**

a) The x-intercept is a point of form (h, 0) on the graph of the equation.

So x = h, y = 0 is a solution of the equation. Substitute in equation and solve for h. 

*That is:* \(4h - 2(0) = 8 \Rightarrow 4h - 0 = 8 \Rightarrow 4h = 8 \Rightarrow h = 2\)

Therefore the x-intercept is the point (2, 0).

The y-intercept is a point of form (0, k) on the graph of the equation.

So x = 0, y = k is a solution of the equation. Substitute in equation and solve for k.

*That is:* \(4(0) - 2k = 8 \Rightarrow 0 - 2k = 8 \Rightarrow -2k = 8 \Rightarrow k = -4\)

Therefore the y-intercept is the point (0, -4).

b) Plot the two points (2, 0) and (0, -4) and draw a straight line through them.
Comment: In practice to find the y-intercept, simply substitute \( x=0 \) in the given linear equation and solve for \( y \). Similarly to find the x-intercept, simply substitute \( y=0 \) in the given linear equation and solve for \( x \).

In applications it is sometimes of interest to know the output when virtually no input (input = 0) has been made. For example the price of an item when supply is almost zero, and this is deduced from the y-intercept. Equally of interest is the level of input that produces no output (output = 0). For example the level of supply at which the price of an item collapses (price = 0), and this is deduced from the x-intercept.

Example 10: (Find the slope and y-intercept from equation)

Write the equation \( 2x - 3y = 6 \) in the standard form \( y = mx + b \).

i) Identify the slope and the y-intercept. ii) Use (i) to plot graph of equation.

Solution

\[ 2x - 3y = 6 \] (Add \(-2x\) to both sides) \( \Rightarrow \) \( 2x - 3y + 2x = 6 + 2x \Rightarrow -3y = 6 + 2x \)

(Divide both sides by \(-3\)) \( \Rightarrow \) \( y = \frac{-2}{-3}x + \frac{6}{-3} \Rightarrow y = \frac{2}{3}x + 2 \) (equation in standard form).

i) By interpretation of equation in standard form: The slope \( m = \frac{2}{3}, \) y-intercept \( = (0, -2) \)

ii) The point \((0, -2)\) is on the graph, plot it. From the value of the slope, a second point on the graph is: \( (0 + 3, -2 + 2) = (3, 0) \), plot it. Draw a line to pass through the two points.

This line is the graph of the equation
Example 11: *(Graph by Slope & y-intercept, or two solutions method)*

Graph the equation $3x + y = 2$.

**Solution**

Method 1: Put equation in the form $y = mx + b$, to find the slope and y-intercept.

$3x + y = 2$ (add $-3x$ to both sides) $\Rightarrow 3x + y + -3x = 2 + -3x \Rightarrow y = -3x + 2$

From the equation: $y = -3x + 2$, the slope $m = -3$, $y - \text{intercept} = (0, 2)$. But $-3 = \frac{-3}{1}$

*The* slope expressed in the form $\frac{-3}{1}$ is used in plotting the graph.

$(0, 2)$ is a point on the graph, and so using the slope, the point $(0 + 1, 2 + -3) = (1, -1)$ is on the graph.

Plot the two points $(0, 2)$ and $(1, -1)$.

Draw a straight line to pass through the two points. This straight line is the graph.

Method 2: Substitute in the equation some numbers for $y$ and solve for $x$, or $x$ and solve for $y$.

Substitute $x = 0$ in equation $\Rightarrow 3(0) + y = 2 \Rightarrow 0 + y = 2 \Rightarrow y = 2$. $x = 0$, $y = 2$ is a solution

Substitute $y = -4$ in equation $\Rightarrow 3x + -4 = 2 \Rightarrow 3x = 6 \Rightarrow x = 2$. $x = 2$, $y = -4$ is a solution

The points $(0, 2)$, and $(2, -4)$ are therefore on the graph.

Plot the two points $(0, 2)$ and $(2, -4)$.

Draw a straight line to pass through the two points. This straight line is the graph.
11.5 Given a Straight line Graph find the Equation

Given the slope and any point on the straight line graph, a second point can be found. A straight line graph is defined by any two points (Euclid), and so also by a slope and a point. Except a straight line graph is given on a coordinate axis, the straight line would therefore be defined by two points, or a slope and a point.

**Methods:**

1. The slope of a straight line is the same for any two points on the line.

That is, if the slope is \( m \) and \((h, k)\) is a given point on the line, then for any other point \((x, y)\) on the line,

\[
\frac{y - k}{x - h} = m.
\]

This equation simplifies to \( y - k = m(x - h) \). So the straight line with slope \( m \) and passing through the point \((h, k)\) is the locus of all points \((x, y)\) such that \( y - k = m(x - h) \) and this is the equation of such a straight line. This is called the point & slope equation.

2. \( y = mx + b \) (\( m \) is slope, \((0, b)\) is \( y \)-intercept) is the standard form of the equation of a straight line. The given two points, or slope and point are used to find \( m \) and \( b \).

**Example 12**

Find the equation of the line which contains the point \((0, 5)\) and has a slope of 2.

**Solution:**

**Method 1:**

Equation of line slope \( m \), containing the point \((h, k)\) is: \( i) \ y - k = m(x - h) \).

From the question: \( m = 2 \), \((h, k) = (0, 5)\). So substitute \( m = 2 \), \( h = 0 \), \( k = 5 \) in \( i) \)

\[
y - 5 = 2(x - 0) \Rightarrow y - 5 = 2x \quad (\text{add 5 to both sides}) \Rightarrow y = 2x + 5 \quad \text{is equation of line}
\]
**Method 2:**

Standard equation is: (i) \( y = mx + b \), \( m \) is the slope, (0, \( b \)) the y-intercept.

From the question: \( m = 2 \), (0, \( b \)) = (0, 5). So substitute \( m = 2 \), \( b = 5 \) in (i).

\( y = 2x + 5 \) is the equation of the line.

**Example 13**

Write the equation of the line containing the points (-4, -5) and (-2, 4).

**Solution:**

Given: Two points (-3, -1) and (2, 4) on a line.

Find: The equation of the line.

*From the two points the slope \( m \) of the line can be found. The question now becomes: find the equation of a line of slope \( m \), and containing (-4, -5) or (-2, 4).*

**Method 1:**

Equation of line slope \( m \), containing the point (h, k) is: (i) \( y - k = m(x - h) \).

From the two points (-4, -5) and (-2, 4) on the line the slope \( m = \frac{4 - (-5)}{-2 - (-4)} \Rightarrow m = \frac{9}{2} \), *and*

using (h, k) = (-4, -5). Substituting in (i) \( \Rightarrow y - (-5) = \frac{9}{2}(x - (-4)) \Rightarrow y + 5 = \frac{9}{2}(x + 4) \Rightarrow y + 5 = \frac{9}{2}x + 18 \) (subtract 5 from both sides) \( \Rightarrow y = \frac{9}{2}x + 13 \)

This is the equation of the line in standard form. To write equation in general form: multiply both sides by 2 \( \Rightarrow 2y = 9x + 26 \Rightarrow 2y - 9x - 26 = 0 \)

**Method 2:**

Standard equation is: (i) \( y = mx + b \), \( m \) is the slope, (0, \( b \)) the y-intercept.

From the two points (-4, -5) and (-2, 4) on the line, the slope \( m = \frac{4 - (-5)}{-2 - (-4)} \Rightarrow m = \frac{9}{2} \) then

substitute this value for \( m \) in (i) \( \Rightarrow (ii) y = \frac{9}{2}x + b \). The value of \( b \) is such that for any point (h, k)

on the line \( k = \frac{9}{2}h + b \). Using (h, k) = (-2, 4) \( \Rightarrow 4 = \frac{9}{2}(-2) + b \Rightarrow 4 = -9 + b \)

(add 9 to both sides) \( \Rightarrow 13 = b \). Substitute \( b = 13 \) in (ii) \( \Rightarrow Equation \) of line is \( y = \frac{9}{2}x + 13 \).
**Method 3:**

Standard equation is: (i) \( y = mx + b \), \( m \) is the slope, \( (0, b) \) the y-intercept. For any point \((h, k)\) on the line; (ii) \( k = mh + b \). The points (-4, -5) and (-2, 4) are on the line.

For the point (-4, -5), substitute \( h = -4 \), and \( k = -5 \) in (ii) \( \Rightarrow (iii) -5 = -4m + b \)

For the point (-2, 4), substitute \( h = -2 \), and \( k = 4 \) in (ii) \( \Rightarrow (iv) 4 = -2m + b \)

Solve equations (iii) and (iv) simultaneously to obtain \( m \), and \( b \). Substitute the values obtained in (i) for the equation of the line.

**Example 14**

The table below shows the length of time in days Maya worked installing solar water heaters in houses and the number of houses with solar water heaters in Brampton.

<table>
<thead>
<tr>
<th>Time in days</th>
<th># of houses with swh</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>16</td>
</tr>
<tr>
<td>8</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>24</td>
</tr>
</tbody>
</table>

i) Write an equation to represent the relation between the number of houses in Brampton with solar water heaters and the length of time in days Maya worked.

ii) How many houses in Brampton had solar water heaters before Maya’s work?

**Solution**

Use values in the table to plot a graph with ‘time in days’ represented by ‘\( t \)’ as input and ‘# of houses with swh’ represented by ‘\( s \)’ as output.

The resulting graph on the right is a straight line. Therefore the relation between \( s \) and \( t \) is linear. It is the equation of the line. (Use any two points e.g. (2, 8) and (6, 16) and apply any of the methods in the previous examples to show that the equation of the line is: \( s = 2t + 4 \).)
Math Skills for Business - Full Chapters

i) \( s = 2t + 4 \) is the relation between the number \( s \) of houses in Brampton with solar water heaters and the length of time in days \( t \) Maya worked installing solar water heaters.

ii) From the equation: for \( t = 0, s = 4 \). So there were 4 houses in Brampton with solar water heaters before Maya started installing them. (Or from the graph the \( s \)-intercept is \((0, 4)\) which leads to the same answer.

The slope, intercepts, and equation of a line are very important concepts with wide application in business. So we advise the student to ensure that she/he has understood and mastered these concepts.

11.6 Review, Exercises and Assignments

In question 1-20, fill in the blanks with the appropriate words or answer true or false.

1. To calculate the rate of change in respect of \( y \) to \( x \) you need a change in ____ and a change in a ____
2. The \( x \)-axis is the ____. That means that it does not depend on anything.
3. The \( y \)-axis is the ____. It depends on the independent variable.
4. In terms of change over time, the \( y \)-value is the ____ axis and the time is the ____ axis.

Answer true or false for #5-#9:

5. The rate of change of the input value to the output value is the same as the slope.
6. The slope of a straight line graph is constant.
7. Calculation of the slope requires only subtraction and then division.
8. For any straight line the rate of change is constant.
9. Non-straight line graphs do not have constant slopes.

10. What two pieces of information are needed to write an equation of a straight line? ________ and ________.

11. A line that slants downward from left to right has a ____ slope.
12. A vertical line has __________________ slope.

13. The formula \( y = mx + b \) is called ____________________________.

14. The y-intercept is ____.

15. The x-intercept is ____.

16. Generally, the dependable variable is usually called ____, and the dependent variable is also called ____.

17. A line that slants upward from left to right has a ____ slope.

18. With a slope and a co-ordinate point we can write an equation of a straight line.

   True or false?

19. A horizontal line has a slope of ____.

20. The slope may also be defined as the rise divided by the run. True or false?

21. The line segment AB has a slope of \( \frac{3}{4} \). If the co-ordinate of point A is (2, 5), the co-ordinates of point B could be which of the following:
   a) (6, 8)  b) (5, 9)  c) (-2, 2)  d. (6, 2)

22. The graph of the equation \( 2x + 6y = 4 \) passes through point (k, 2). What is the value of k?

23. Point (-3, h) lies on the line whose equation is \( x - 2y = -2 \). What is the value of h?

24. Which statement best describes the graph of \( x = 4 \)?
   a) It passes through the point (0, 4)
   b) It has a slope of 4.
   c) It is parallel to the y-axis.
   d) It is parallel to the x-axis.

25. What is the y-intercept of the graph of the line whose equation is \( y = \frac{-2}{5} x + 4 \)?

26. If point (-1, 0) is on the line whose equation is \( y = 2x + b \), what is the value of b?
27. If the value of the input variable $y$ increases as the value of the independent variable $x$ increases, the graph of this relationship could be
a) Horizontal line    b) Vertical line    c) One with a negative slope   d) One with a positive slope.

28. The line $3x - 2y = 12$. Which of the following is correct?
 a). A slope of $\frac{3}{2}$ and a y-intercept of -6.
 b) A slope of $-\frac{3}{2}$ and a y-intercept of 6.
 3) A slope of 3 and a y-intercept of -2 
 d) A slope of -3 and a y-intercept of -6

29. Write an equation of the line that has a slope of 3 and a y-intercept of -2

30. What is the x-intercept of the line $2x - 3y = 12$?

31. The data in the table shows the cost (in $) of renting a bicycle in a resort by time (time rented in hours), including deposit.

<table>
<thead>
<tr>
<th>Hours (time rented)</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>8</td>
<td>45</td>
</tr>
</tbody>
</table>

a) What would be the equation of a line that fits the data?

b) What is the amount of the deposit?
32. The equation of line A is $5x + 6y = 3$, and the equation of line B is $5x - 6y = 3$.

Which of the statement below about the two lines is true?

a) Lines A and B have the same y-intercept.

b) Lines A and B have the same slope.

c) Lines A and B have the same x-intercept.

33. Write the equation of each line containing,

a. $A_1(3,2)$ and $A_2(3, -4)$

b. $B_1(2, 1)$ and $B_2(-2, -3)$

c. $C_1(-3, 5)$ and $C_2(-2, 3)$

d. $D_1(-1, -2)$ and $D_2(3, -2)$

34. Write the equation of each line that has the specified slope and contains the given points.

a) Point $(2, -3)$, $m = 3$  

b) Point $(0, -3)$, $m = -1$

c) Point $(3, 5)$, $m = -\frac{3}{7}$  

d) Point $(5, 1)$, $m = -\frac{4}{5}$

35. Graph the lines that passes through point

a. $(-2, -3)$ and has a slope $\frac{5}{4}$

b. $(2, 0)$ and has a slope -1

36. Calculate the slope and y-intercept of each of the equations below.

a. $2x - 3y = 6$  

b) $x - 3y = 3$

c) $4x - 5y = 5$  

d) $4x + y = 2$

37. Find the slope of each line

a. Contains points $(-2, 3)$ and $(4, 2)$

b. Contains points $(2, 5)$ and $(-2, 5)$
38. Using the equation \( y = -\frac{3}{4}x + 2 \), find \( y \) when \( x = 3 \).

39. Graph \( 2x + 3y = 6 \) by using the \( x \)- and \( y \)-intercepts.

40. Intra Manufacturing Inc makes wood-burning heaters for rural and remote population. The production cost and quantities produced are linearly related. It costs $1500 to make 20 heaters and $2100 to make 30 heaters.
   a) If \( C \) is the cost of making \( x \) heaters, write an equation for this relation.
   b) Use the equation to calculate the cost of making 35 wood-burning heaters.

41. In 2000, the total gross sales of Ridgeview Electronic were $350000. The sales were $400000 in 2001 and $450,000 in 2002. Let \( s \) represent the total sales in \( x \) years.
   a. List the three co-ordinate points.
   b. Plot a graph relating \( s \) and \( x \).

42. The Nanest Company produces solar panels. The analysis of the relations between the production cost (\( C \)) of the company and its production quantities (\( Q \)), is described using the following linear equation:
   \[ C = 1500 + 300Q \]
   a) Identify both the slope of the equation and its \( c \)-intercept.
   b) In a particular week, the production quantities were 500 solar panels, what was the production cost?
   c) If production cost was $45000, how many quantities were produced?
   d) What is the unit cost of producing 200 quantities?

43. For each of the following linear equations, list three solutions as ordered pairs in the form (\( x, y \)).
a) \( y = x + 2 \) 

b) \( y = 2x \) 

c) \( y = 3x - 1 \)

44. a) Find four ordered pairs \((a, b)\) for the formula, \(b = a + 3\), for \(a = 0, 1, 2\) and \(5\).

b) Draw the graph using the five ordered pairs in (a).

c) Draw a line through the points.

45. The cost (in dollars) of producing a college newspaper is given by using the Formula, \( C = 2n + 400 \), where \(n\) is the number of copies printed and \(C\) the total cost.

a. What is the slope of the formula?

b. Draw a graph for the formula. (Hint: use \(n = 100, 200, 300,\) and \(500\))

c. Use the graph to estimate the cost of producing thousand copies.

46. a) Draw the graph of a straight line passing through the points \((0, 0)\) and \((3, 6)\).

b) Find the slope of the line.

c) Write an equation for the line.

47. A certain car is expected to depreciate in value according to the equation,

\[ y = -2000x + 40000 \]

where \(y\) is the value of the car (in $) and \(x\) the age of the car (in years)

a) Find the slope of the line and interpret its meaning.

b) Find the \(y\)-intercept and explain what it means.

48. Write the equation of the line that:

a) passes through the point \((3, 4)\) and has slope \(-3\)

b) passes through the point \((-1, -4)\) and has slope \(\frac{1}{2}\).

49. What is the slope of the graph of the linear equation \(5y - 10x - 15 = 0\)?
50. The table below shows the enrolment of a daycare centre from 2002 through 2006.

<table>
<thead>
<tr>
<th>Year(x)</th>
<th>Enrolment (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002</td>
<td>14</td>
</tr>
<tr>
<td>2003</td>
<td>20</td>
</tr>
<tr>
<td>2004</td>
<td>22</td>
</tr>
<tr>
<td>2005</td>
<td>28</td>
</tr>
<tr>
<td>2006</td>
<td>37</td>
</tr>
</tbody>
</table>

a) Draw a graph for the above relations
b) Between 2002 and 2006, enrolment increased by what percent?
c) Which year had the lowest increase in enrolment?
d) Which year had the highest increase in enrolment?

51. A straight line was created using the following data.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-6</td>
<td>-3</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

What are the x-and y-intercept for this line?

52. A and b are linearly related. The values of a and its corresponding values are given in the table below:

<table>
<thead>
<tr>
<th>a</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

Write a formula for this relation.
53. Given the equation \( y = x - 2 \).
   a) What is the slope of this equation?
   b) Does the graph of this equation rise or fall from left to right?
   c) What is the \( y \)-intercept?
   d) What is the \( x \)-intercept?
   e) Now graph the equation.

54. Imagine the graph of the following equation:
   \[ 4x - 5y + 20 = 0 \]
   What is the slope of the line?

55. The equation \( C = 0.05t + 10 \), represents the relations between the total cost (in $) \( C \), charged by an internet service provider, and time (in hours) \( t \), used.
   a) What is the slope of the equation?
   b) Explain the meaning of the slope within the context of the equation.
   c) What is the \( y \)-intercept? What does it mean?

56. Student painters charge $5.00 per square metre plus an additional fee of $25.00 to paint a living room.
   a) Graph showing the relations between the fee charged and the area painted.
   b) Use your graph to estimate the charge for a living-room which is 25 \( m^2 \).
57. Kid’s Party Place charges $20 for a party room plus $12 for each person attending. The chart below shows the total cost for 10, 15, and 20 people attending a party.

<table>
<thead>
<tr>
<th>Number in Attendance</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>$140</td>
</tr>
<tr>
<td>15</td>
<td>$200</td>
</tr>
<tr>
<td>20</td>
<td>$260</td>
</tr>
</tbody>
</table>

a) Graph the above information.
b) Calculate the slope for the line.
c) Write an equation for the line

58. The data below represents the percent growth in the net profit of a company from year 2000 to 2004.

<table>
<thead>
<tr>
<th>Year</th>
<th>Percent (%) Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>20.00</td>
</tr>
<tr>
<td>2001</td>
<td>21.50</td>
</tr>
<tr>
<td>2002</td>
<td>23.00</td>
</tr>
<tr>
<td>2003</td>
<td>24.50</td>
</tr>
<tr>
<td>2004</td>
<td>25.00</td>
</tr>
</tbody>
</table>

a) Plot a graph showing the company’s net profit growth.
b) Calculate the slope of the line.
12.1 What is a System of Equations?

In section three on equations, you solved equations involving one variable or unknown or placeholder. There are some situations in which two or more variables are involved. Such situations entail two sets of combination of things. However, in this section, we will restrict ourselves only to those involving two variables. A problem that requires two equations to be constructed in order to solve a problem is called a system of two linear equations in two variables. Some mathematicians call this system simultaneous equation because of solving the two equations together rather than separately. The example below illustrates the nature and characteristics of simultaneous equations.

Example

Suppose you have $75 for shopping. You could buy 2 CDs and 3 blank cassettes with the money. You could also buy 1 CD and 9 blank cassettes with the money. If the cost of CD is \( d \) and the cost of a blank cassette is \( c \), find the cost of one CD and the cost of one cassette.

Solution

Let us construct two separate equations.

Since \( d \) represents CD and \( C \) represents cassette, we have

\[
2d + 3c = 75 \Rightarrow \text{Equation 1}
\]

\[
1d + 9c = 75 \Rightarrow \text{Equation 2}
\]

We need a pair of numbers that is a solution to both equations. So let us draw a table of numbers for \( 2d + 3c \) and \( 1d + 9c \), using different numbers for \( d \) and \( c \). We simply substitute each number for \( d \) and \( c \) in the equations to see if we give exactly $75.00.

<table>
<thead>
<tr>
<th>d</th>
<th>5</th>
<th>7</th>
<th>8</th>
<th>10</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>6</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>( 2d + 3c ) = $75</td>
<td>46</td>
<td>50</td>
<td>28</td>
<td>38</td>
<td>58</td>
<td>75</td>
</tr>
<tr>
<td>( 1d + 9c ) = $75</td>
<td>113</td>
<td>115</td>
<td>62</td>
<td>64</td>
<td>74</td>
<td>75</td>
</tr>
</tbody>
</table>
You will find that in the table there is a pair of numbers for \( d \) and \( c \) that satisfies both equations. This pair of numbers, \( d = 30 \) and \( c = 5 \) is a solution for both equations simultaneously. Because, \( 2(30) + 3(5) = 75.00 \), and \( 1(30) + 9(5) = 75.00 \), that pair is the solution to the two equations. One CD costs $30.00 and one cassette costs $5.

We have to repeat the point that to solve a linear equation with two unknowns, two different but related equations are needed. We also have to caution the student that not all simultaneous equations have solutions. When solving any system of equations, you should know that:

- d) the system may have no solution;
- e) the system may have one system (one number each variable);
- f) The system may have infinitely many solutions.

The procedure of trial and error that we used to solve the problem in example 1 is not most appropriate. This is because it is time consuming as it involves trying several pairs of numbers until we find the pair that satisfies both equations. As well, the procedure is cumbersome and one is more likely to make many mistakes. We need a better method that is more reliable, simpler, and economical in time.

### 12.2 Solving a system of Equations

Over the years mathematicians have invented three methods for solving a system of equations in two variables. These are elimination (or addition) method, substitution method, and graphical method. The use of any of the first two methods to solve a system of equations depends on the nature of the problem. Some problems are easily solved using elimination rather than substitution. However, the graphical method, despite that it gives only approximated solution could be used to solve any systems of equations. Though three methods or variations of them are customarily used to solve a system of equations, it does not mean that the student should not invent his or her own method for solving such a system. We discuss each of these three formal methods with illustrations.
i) Elimination (Addition) Method

With this method, one of the variables is eliminated through a process of cancellation. And then the equation is solved for the remaining variable. The number for the remaining variable is substituted in the original equation to solve for the eliminated variable. To eliminate a variable, you must obtain a variable in the other equation that differs from the other in sign. Alternatively, one equation may be subtracted from or added to the other in order to eliminate one variable.

Example 1

What is the solution in the form \((x, y)\) for the following system of equations?

\[
4x + 3y = 26 \\
2x - y = 8
\]

Solution

In this example, the two equations have been constructed already. Our task is to solve it and write the answer in the ordered pair form \((x, y)\).

\[
\begin{align*}
4x + 3y &= 26 \quad \Rightarrow \text{Equation (1)} \\
2x - y &= 8 \quad \Rightarrow \text{Equation (2)}
\end{align*}
\]

If you examine the two equations carefully, you may see that if we multiply each term of equation 2 by 3, we get \(2x \cdot 3 - 3 = 8 \cdot 3\). By adding equation 1 and 2 together, we are able to eliminate \(y\).

\[
\begin{align*}
4x + 3y &= 26 \\
6x - 3y &= 24
\end{align*}
\]

Adding these, \(10x = 50\)

\(x = 5\)

To find \(y\), substitute 5 into \(x\) in any of the original equations.

\[
\begin{align*}
2(5) - y &= 8 \\
10 - y &= 8 \\
y &= 10 - 8 = 2
\end{align*}
\]

The ordered pair \((x, y) = (5, 2)\)
#### Example 2

Mountain Ridge Restaurant is having lunch special. It is offering 2 slices of pizza and 1 pop (a choice of coke, Pepsi, spirit, C-plus, etc.) for $3.50 or 3 slices of pizza and 2 pops for $5.75. How much is 1 slice of pizza and 1 pop?

**Solution**

We have two separate situations and they require two separate equations. Let us take p for pizza and q for pop. Then, we have the following equations to solve.

1. \(2p + q = 3.50\) \(\Rightarrow\) Equation (1)
2. \(3p + 2q = 5.75\) \(\Rightarrow\) Equation (2)

Ask yourself which variable is easier to eliminate? Certainly we can eliminate q, since it is a lone variable in the first equation. To eliminate q, let us multiply the first equation by 2 and subtract the 2 equation from equation 1. We have,

\[
\begin{align*}
2p + q &= 3.50 \quad \text{Equation (1)} \\
3p + 2q &= 5.75 \quad \text{Equation (2)}
\end{align*}
\]

Subtracting, \(p = 1.25\)

So each slice of pizza costs $1.25. Let us solve for q by substituting 1.25 for p in the original equation. We have,

\[
\begin{align*}
3p + 2q &= 5.75 \\
3(1.25) + 2q &= 5.75 \\
3.75 + 2q &= 5.75 - 3.75 \\
2q &= 2.00 \\
q &= 1.00
\end{align*}
\]

So a pop costs $1.00.

#### Example 3

Eight thousand people attended a Christmas concert in a stadium. The ticket prices were $50 for an adult and $30 for a child under 18 years old. The total revenue from the ticket sales was $250,000. The concert promoters want to know this vital information for the purpose of future planning: How many tickets of each were sold?
Solution

After reading the problem, you will know that there are two categories of attendees at the concert: adults and children. The price for each is different, and the number of tickets sold for each category may also be different. To know how many tickets of each category were sold, we have to construct two equations. Let A be the number of adults who attended, and C be the number of children who attended.

\[(\text{# of adults}) + C (\text{# of children}) = 8000 \text{ (Total number of attendees)}\]

\[
\begin{align*}
A &+ C = 800 \\
\downarrow &+ \downarrow &\downarrow \\
A &+ C = 800
\end{align*}
\]

We also know that,

\[(\text{# of adults x } $50) + (\text{# of children x } 30) = 250000 \text{ (Total receipts)}\]

\[
\begin{align*}
50A &+ 30C = 250000
\end{align*}
\]

Thus we have to two sets of equations:

\[
\begin{align*}
A &+ C = 8000 \Rightarrow \text{Equation 1} \\
50A &+ 30C = 250000 \Rightarrow \text{Equation 2}
\end{align*}
\]

We can choose to eliminate any of the variables. Let us say we want to eliminate A. We have to multiply the first equation by a number such that when we subtract equation (2) from equation (1), A will be eliminated and we will be left with only C to solve. Let us multiply equation (1) by 50 and subtract equation (2) from it. We have,

\[
\begin{align*}
50A &+ 50C = 400000 \\
50A &+ 30C = 250000 \\
20C = 150000 \Rightarrow \text{Result for subtracting equation (2) from equation (1).} \\
C = 7500 \Rightarrow \text{Result for dividing both sides by 20.}
\end{align*}
\]

A total of 7500 children attended. Since 8000 attended we know that 500 adults attended \((8000 - 7500)\).

So 7500 children and 500 adults attended.
Example 4

Redco Inc. Manufactures both 8” portable DVD player and 10” portable DVD player. The material cost per 10” portable DVD player is $35, while the material cost for the 8” portable DVD player is $25. The labour cost for manufacturing a 10” portable DVD player is $40 and that of 8” portable DVD is $20. The company has a weekly material budget of $1550 and labour cost budget of $1600. How many portable DVD of each dimension does the company plan to manufacture every week?

Solution

From the information provided, we know that the cost of making any of the portable DVDs has two components: material and labour. We also know that the company has a weekly material budget of $1550 and labour for $1600 for both products.

Let T stand for the number of portable DVD players that will be manufactured.

Let E stand for the number of 8” portable DVD players that will be manufactured.

So we have,

\[ 35T + 25E = 1550 \]  \( \text{The material cost for both products} \)

\[ 40T + 20E = 1600 \]  \( \text{The labour cost for both products} \)

Now that you have two equations, think about which of the variables you want to eliminate. Let us say we want to eliminate E. It is the easier one, since we can multiply the first equation by -4 and the second equation by 5 and add both equations together. Thus, we have,

\[ -4 \times (35T + 25E = 1550) \Rightarrow -140T - 100E = -6200 \]

\[ 5 \times (40T + 20E = 1600) \Rightarrow 200T + 100E = 8000 \]

\[ 60T = 1800 \Rightarrow \text{The result after adding.} \]

\[ T = 30. \text{ The company plans to make only 30 of 10” portable DVD for a week} \]

To find how many of 8” portables it plans to make use one of the original equations and substitute \( T = 30 \). Let us use equation (1).

\[ 35T + 25E = 1550 \]

\[ 35 (30) + 25 = 1550 \]
The company will produce 20 of 8” portable DVD.

We summarize the steps used to solve a system of equation by elimination.

Inspect the equations carefully to see if by adding or subtracting will help you to eliminate one of the variables.

If this not possible, think of a number that when you multiply by each term of one equation or both will enable you to eliminate one of the variables.

When one variable is eliminated through addition or subtraction, solve the equation for the remaining variable.

Plug the value for that variable into one of the original equations and solve for the variable you eliminated in either step one or two.

ii) Substitution Method

This method works when one of the equations has a lone variable on one side of that equation. In that case, solve for lone variable. That equation must be substituted into the other equation, combined to form a new equation. And then solve for the remaining variable.

Example 5

We use the same question in example 2.

What is the solution in the form \((x, y)\) for the following system of equation?

\[
4x + 3y = 26
\]

\[
2x - y = 8
\]
Solution

There is a lone variable, \(y\), on the left side of the second equation. So we solve for \(y\) in that equation.

\[2x - y = 8\]

\[-y = 8 - 2x \Rightarrow \text{Multiply each side by -1.}\]

\[y = 2x - 8\]

\(\Rightarrow\) Substitute this equation into the first equation for \(y\) in the other equation.

\[4x + 3y = 26\]

\[4x + 3(2x - 8) = 26 \Rightarrow \text{Remove the bracket by multiplying each term by 3.}\]

\[4x + 6x - 24 = 26 \Rightarrow \text{simplify by combining like terms.}\]

\[10x - 24 + 24 = 26 + 24 \Rightarrow \text{Add 24 to each of the equation to isolate 10x.}\]

\[10x = 50 \Rightarrow \text{Divide each side by 10}\]

\[x = 5\]

Let’s solve for \(y\), using one of the original equations. Remember \(x = 5\)

\[4x + 3y = 26\]

\[4(5) + 3y = 26\]

\[20 + 3y = 26 - 20 \Rightarrow \text{Add -20 to both sides to isolate 3y.}\]

\[3y = 6 \Rightarrow \text{Divide each side by 3}\]

\[y = 2\]

Example 6

The tickets for a community dance recital cost $5 for adults and $2.00 for children and adolescents. The total number of tickets sold was 295 and the total amount of money collected was $1,220. How many adult tickets were sold?

Solution

Two situations are presented: # of adult tickets sold and # of children tickets, and total amount for adult for adult tickets sold and that for children tickets sold.
Let \( A \) represent the number of adult ticket sold, and \( C \) the number of children ticket sold. This gives us this equation, \( A + C = 295 \).

When we multiply the number of tickets for adults by the price of the ticket, and do the same for the children we get the following equation.

\[
(A \times 5) + (C \times 2) = 1,220
\]

So we have two equations:

\[
\begin{align*}
A + C &= 295 \quad \Rightarrow \quad (1) \\
5A + 2C &= 1,220 \quad \Rightarrow \quad (2)
\end{align*}
\]

The two variables are alone in the left side in equation (1). So let us solve that equation in terms of \( A \) and substitute the result in \( A \) in equation (2). We have, \( A = 295 - C \).

\[
\begin{align*}
5A + 2C &= 1,220 \quad \Rightarrow \quad (2) \\
5(295 - C) + 2C &= 1,220 \quad \Rightarrow \quad \text{Remove the bracket by multiplying each term by 5.} \\
1475 - 5C + 2C &= 1220 \quad \Rightarrow \quad \text{Simplify} \\
1475 - 1475 - 3C &= 1220 - 1475 \quad \Rightarrow \quad \text{Add -1475 to each side of the equation.} \\
-3C &= -255 \quad \Rightarrow \quad \text{We can not have a negative C so multiply each by -1.} \\
\frac{-3C}{3} &= \frac{255}{3} \quad \Rightarrow \quad \text{Divide each side by 3.} \\
C &= 85
\end{align*}
\]

So 85 children attended the community concert or 85 children tickets were sold.

To solve for \( A \), substitute \( C=85 \) in one of the original equations. We have

\[
\begin{align*}
A + C &= 295 \\
A + 85 &= 295 \\
A &= 295 - 85 \\
\end{align*}
\]

Thus 210 adult tickets were sold.
We can check our solution in the original equation, as we did in section three. Let us do this together by substituting \( A = 210 \) and \( C = 85 \)

\[
\begin{align*}
A + C &= 295 \quad \Rightarrow (1) \\
5A + 2C &= 1,220 \quad \Rightarrow (2)
\end{align*}
\]

\[
\begin{align*}
210 + 85 &= 295 \quad \Rightarrow (1) \\
5(210) + 2(85) &= 1,220 \quad \Rightarrow (2)
\end{align*}
\]

Note that example 6 and 3 are similar. You might have noticed that in each case after setting up the equations, there were lone variables on the left side of the equation. This makes both examples 6 and 3 easily solvable by substitution method. So if a variable can easily be solved \textit{in terms of the other}, a system of simultaneous equation is solvable by substitution method. Otherwise, it is better, to make calculation easier, to use the elimination method. This condition, as you will see below, is also applicable to the graphical method.

\textbf{iii) Graphical Method}

In this method, each equation is graphed on the same plane. However, it is not important which of data is graphed on the horizontal axis or the vertical axis. After graphing two equations, the ordered pair of the point where the two lines intersect is the solution of the system of equation. That is, the \( x \) and \( y \) values where the lines intersect are the solution to the equations.

Note that when solving a system of simultaneous equations by graphical method, any three of the following possibilities is likely to arise:

1) The lines cross just once. In this case, there is only one number for each of the variables.

2) The lines never cross. This case occurs where two lines are parallel and do not touch each other. Parallel lines have the same slope.

3) The lines lie on top of each other. When this case happens there are an infinite number of \((x, y)\) pairs that will satisfy both equations.
Indeed we can check the number of solutions to a system of simultaneous equation by rearranging both equations in the slope-intercept form \( y = mx + b \), and then compare the slopes (or gradients) and the \( y \) intercepts. If the slopes are different, we will have a single solution. If the slopes are the same but the intercepts are different, we will have no solution to the system. Finally, if both the slopes and the \( y \)-intercepts are the same, then there will be many solutions to the system. We summarize these features below. Remember from Chapter 10 that the symbol for the slope is \( m \) and for the \( y \)-intercept is \( b \).

- If \( m_1 \neq m_2 \) then there is only one solution to the system.
- If \( m_1 = m_2 \) and \( b_1 \neq b_2 \), there is no solution to the system.
- If \( m_1 = m_2 \), and \( b_1 = b_2 \), there are many solutions to the system.

We now illustrate how to solve a system of simultaneous equations using graphical method.

**Example 7**

Solve the following system of simultaneous equation:

\[
\begin{align*}
  x + y &= 2 \\
  x - y &= 1
\end{align*}
\]

**Solution**

First, we generate three ordered pairs for each by plugging any number of our choice. However, the sensible thing to do is to calculate the intercepts- both \( x \) and \( y \).

Let us start with equation (1).

To find the \( x \)-intercept, let \( y = 0 \), then we have, \( x + 0 = 2 \). That is, \( x = 2 \). The ordered pair is \( (2, 0) \).

To find the \( y \)-intercept, let \( x = 0 \), then we have, \( 0 + y = 2 \). That is, \( y = 2 \). The ordered pair is \( (0, 2) \).

Let us generate two more ordered pairs. Let \( x = 1 \), then \( 1 + y = 2 \); \( y = 1 \). The ordered pair is \( (1, 1) \). Let \( x = 2 \), \( 2 + y = 1 \); \( y = -1 \). The ordered pair is \( (2, -1) \).
Let us do the same for equation (2), \( x - y = 1 \).

The \( y \)-intercept is (0, -1), and the \( x \)-intercept is (0, 1). We generate two more ordered pairs. Let \( x = 2 \), \( 2 + y = 1 \); \( y = -1 \). Ordered pair is (2, -1). Let \( x = 1 \), another ordered pair is (1, 0). We plot the graph below.

Example 8

Coffee-In sells 6 cups of tea and 4 pieces of cookie for $5.30, while 4 cups of tea and 2 pieces of cookie for $3.40. How much is one cup of tea and a piece of cookie?

Solution

Before we write the equations, let us have \( t \) represent tea, and \( c \) for cookies. Then we have the following equations.

\[ 6t + 4c = 5.30 \Rightarrow (1) \]

\[ 4t + 2c = 3.40 \Rightarrow (2) \]

Second, we have to decide which axis we are going to graph the information about tea and cookies. Let us assume that the purchase of cookies depend on tea. Thus the
information about the tea will be displayed on the x-axis and those for the cookies will be on the y-axis.

Third, we need to choose a scale for each axis. To avoid any difficulties, let us have one square represent $2.00 on both axes. So half a square represents $1.00 and a quarter square stands for 25cents.

Fourth, we need at most three ordered pairs for each equation before we can do the graph. We start with equation (1)

\[ 6t + 4c = 5.30 \]

Let \( t = 0.40 \).

We have, \( 6 (0.40) + 4c = 5.30 \)

\[ 2.40 + 4c = 5.30 \]
\[ 4c = 2.90 \]
\[ c = 0.73 \]

The ordered pair is \((0.40, 0.73)\).

Let us use the intercepts.

To find the x-intercept, \( c = 0 \)

\[ 6(0) + 4c = 5.30 \]
\[ 6t = 5.30 \]
\[ t = 0.88 \]

The ordered pair = \((0.88, 0)\).

We do the same for equation (2), \( 4t + 2c = 3.40 \)

Let us find the intercepts. Let \( c = 0 \), \( 4t + 2(0) = 3.40 \)

Let \( t = 0 \).

\[ 4t = 3.40 \]
\[ 4(0) + 2c = 3.40 \]
\[ t = 0.85 \]
\[ 2c = 3.40 \]
\[ c = 1.70 \]

The ordered pair is \((0.85, 0)\).

Let \( t = 0.70 \), then \( 4(0.70) + 2c = 3.40 \)

\[ + 2c = 3.40 \]
\[ 2c = 0.60 = 0.30 \] 
The pair is (0.70, 0.30) 

We now have enough ordered pair to draw the graph. But before we do so, note that graphical solution is approximation; more so when cents are involved.

We summarize the steps for solving a system of equation using the method of substitution.

- Solve one of the equations in terms of one of the variables.
- Plug the new equation into the other equation.
- Solve for the equation to find the value for the other variable.

12.3 Which Method is Appropriate?

We conclude this section with the above question. The answer is this: it depends on the nature of the system of simultaneous equation you are trying to solve. In general, the following are guidelines are helpful.

1. If there is a lone variable on any side of either equation, substitution is a better method.
2. If you can easily solve one of the equations in terms of the other variable without ending with fractions or decimals, then substitution is a better method.
3. If approximation is okay and time is not an issue, then the graphical method may used to solve the system.
4. If accuracy is an issue then using the algebraic methods of elimination or substitution are better than the graphical method.

Having said all that, let us say that using any of the methods you find attractive and convenient to solve a system of simultaneous equation is fine. However, the most important factors you have to remember in solving a problem involving a system of equation are as follows: 1) identify two unknowns by reading the problem thoroughly
and label them with letters, 2) Set up a system of two equations, 3) solve the equations simultaneously, 4) Check your solutions in the original equations, 5) Ask yourself if the solutions make sense, 6) Read the problem one more time and relate it to the solutions you have provided.

12.4 Review, Exercises, and Assignments

1) Mary invested $40,000 in two separate investments with a bank. One of the investments pays 5% interest and the other 8%. The total interest earned on the two investments was $2,500. How much did she invest at each interest rate?

2. Solve graphically each of the following system of equations:
   a) \[ 5x + 2y = -4 \]
   b) \[ x + 3y = 7 \]
   c) \[ y = -3x + 2 \]
   \[ 2x + 3y = -6 \]
   \[ -x + 3y = 5 \]
   \[ y = 2x - 8 \]

3. Ricardo is a building contractor. If he hires 8 carpenters and 2 painters, his total daily payroll amounts to $960, while hiring 10 carpenters and 5 painters cost him $1500 daily. How much does it cost him to hire one carpenter and one painter?

4. Ray is a vegetable farmer. He has a total of 250 plants of sweet pepper and green beans. Each sweet pepper plant yields 3kg of pepper and each green bean plant yields 4kg of green beans. Ray’s total harvest of the two vegetables was 810 kg. How many green beans plants does Ray have on his farm?

5. Six nights at Club Resort, including plane fare cost $980. Nine nights at the Club Resort, including plane fare cost $1,235. For both holiday packages, the plane fare is the same as well as the charge for accommodation per night. Find the plane fare and the charge for accommodation per night.
6. A drink and a bag of potato chips cost $1.30. Two drinks and a bag of potato chips cost $2.10. Which of the following could Andrea buy with $5.00?
   i) A drink and 8 bags of potato chips
   ii) 2 drinks and 7 bags of potato chips
   iii) 3 drinks and 6 bags of potato chips
   iv) 4 drinks and 4 bags of potato chips

7. Fun Travel Agency offers the following holiday packages.

<table>
<thead>
<tr>
<th>Package 1</th>
<th>Package 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1200 for 5 nights, including</td>
<td>$1400 for 7 nights, including</td>
</tr>
<tr>
<td>round-trip plane fare and hotel</td>
<td>round-trip plane fare and hotel</td>
</tr>
<tr>
<td>accommodation.</td>
<td>accommodation.</td>
</tr>
</tbody>
</table>

The length of stay does not affect the plane fare and charge for accommodation.

a) What is the cost of the round-trip plane fare?
b) How much is the charge for hotel accommodation per night?

8. Solve each of the following system of equations by elimination.

   a) \[ x + y = 5 \]
   b) \[ 3x + 2y = 18 \]
   c) \[ 2x + 5y = 2 \]

   \[ x - y = 3 \]
   \[ x + y = 1 \]
   \[ 3x - 2y = -16 \]

9. A golf club charges its members a fixed annual fee and a green fee for each golf game played. In one year, Nguyen played 13 games and paid $811. In the same year, Asiatou played 28 games and paid $1,189.

   a) How much was the annual fee?  
   B) How much is the green fee?
10. There are two parking lots on a university campus. The north-end parking-lot charges a flat fee of $10 and $0.50 per hour for parking. The south-end charges a flat fee of $5 and $1.50 per hour for parking.
   a) For each parking lot, write an equation that represents the total cost for parking.
   b) Graph each of the two equations on the same plane.
   c) Using the graph find the length of time of parking in hours for which the cost for parking would be the same for both parking lots.

11. Mark and Yan own separate mowing businesses. Mark’s profit equation for mowing this year is \( P = 15n - 300 \), whilst Yan’s is \( P = 20n - 450 \); where \( P \) is profit and \( n \) is the number of lawns mowed.
   a) What could be the meaning of -300 and the -450 in the equations respectively?
   b) Solve the system of profit equations.
   What does the solution mean in the context of the problem? Explain your answer.

12. A leather jacket and two dress shirts cost $270. The jacket costs $80 more than the cost of a shirt.
   a) How much does the jacket cost?
   b) How much does one shirt cost?

13. At a gas station, Nadia paid $37.50 for a can of engine oil and 25 litres of petrol. Maggie paid $26.40 for 2 cans of the same type of engine oil and 15 litres of the same type of petrol. At what price is the gas station selling a can of this type of engine oil and a litre of this type of petrol?
14. What are the advantages and disadvantages of each of the three methods for solving a system of simultaneous equations?

15. Four pens and one calculator cost $54. Two pens and 2 calculators cost $72.
   a) Write a pair of equations for this situation.
   b) Find the price of a pen and a calculator. (State any assumptions made)

16. Igor purchased two donuts and three resin cookies at Breakfast Lane for $3.30. Paul purchased five donuts and two resin cookies at the same place for $4.95. All the donuts have the same price and all the cookies have the same price.
   Find the cost of one donut and the cost of one resin cookie.

17. Compare the substitution method and graphical method. What are the pros and cons of each method?

18. At a circus fair, the cost of a ride is $1.00 per child and $2.00 for each adult. The Simpson family had a total of 11 rides and spent $14.00. How many of each type did they ride?

19. What is the solution to the following system of simultaneous equation?
   a) \(3x - y = 8\)  
   x + y = 2
   b) \(y = -3x - 2\)
   6x + 2y = -4

20. Which ordered pair is the solution to the system of equation below?
   x + 3y = 7
   x + 2y = 10
   A. \((\frac{7}{2}, \frac{13}{4})\)  
   B. \((\frac{7}{2}, \frac{17}{2})\)
   C. (-2, 3)  
   D. (16, -3)
21. It takes a machine one hour to produce 3 units of product A. It takes the same machine one hour to produce 2 units of product B. In August 2008, the company budgeted 100 hours of machine time anticipating to produce a total of 220 units of both products.

   a) How many units of both products can the company produce?
   b) What factors should the company consider in budgeting for machine hours?

22. Solve the following system of equation.
   \[ 2x + 8y = 14 \]
   \[ x + 4y = 7 \]

23. Ama owns a flower shop. Her charge for a bouquet of roses is made up of the cost of the vase plus the cost per rose. She charges $32.85 for a bouquet of 12 roses, and $50.85 for a bouquet of 20 roses. How much does she charge for a vase? Assume charge for a vase does not change with size of bouquet.

24. Plot the graphs of \( x + y = 2 \) and \( y - 2x = 5 \) on the same axis. Use your graphs to solve the simultaneous equation
   \[ x + y = 2 \]
   \[ y - 2x = 5 \]

25. Without solving the system of equations below, how many solutions would each have? (Write each equation in slope-intercept form.)
   a) \( 2y = 6x -4 \)  \( y + x - 2 = 0 \)
   b) \( y = 3x -2 \)  \( y - x + 1 = 0 \)

Check your answers by graphing each pair of equations.
26. Freddy is a home-owner. During the month of June, he used 500 units of hydro and 100 units of natural gas for a total of $176. The following month, July, he used 400 units of hydro and 150 units of natural gas for a total of $152. Find the cost per unit of hydro, and the cost per unit of natural gas.

27. A building contractor purchased 80 metres of redwood and 100 metres of pine for $54. A second purchase, at the same prices per metre, consisted of 100 metres of redwood and 80 metres of pine for $72. Find the cost per metre of the redwood and pine.

28. A company manufactures both 20” HDTV sets and standard 20” TV sets. The material cost for manufacturing a standard TV set is $40 and $100 for HD TV set. The labour cost for the manufacture of standard TV set is $30, and $60 for a HDTV set. The company has a weekly budget of $4500 for material and $3100 for labour. How many of each type of TV does the company plan to manufacture a week?

29. If \( x + y = 9x + y \), then \( x \) is equal to what value?

30. Super Mart is having a sale of CDs. The CDs are on sale for $15 and $20 each. Steven spent $180 to buy some of the CDs. He bought two more of the expensive ones than the cheaper ones. How many of each did Steven buy?

31. A Community Association sold 500 tickets for its annual end of year concert. It sold as many as 4 times the tickets at the door than it sold in advance. An advance ticket cost $30 and tickets at the door cost $35. How much money was collected for the end of year concert?
32. Judy and Linda went for lunch at a shopping centre. Judy ordered three slices of pizza and two cans of Pepsi. Linda ordered two slices of pizza and one can of Pepsi. Judy’s bill was $10.00 and Linda’s bill was $6.00.

a) What was the price of one slice of pizza?
b) What was the price of one can of Pepsi?

U4 Full Chapter (continued)

9-Problem solving: Process, and Strategies

9.1 Introduction to Problem-Solving

This course has two major objectives: to help you develop mathematics skills and knowledge, and apply those skills and knowledge to solve mathematical problems in a business environment.

Problem solving is an integral part of our lives. We solve problems every day such as cooking a specific food we want to eat, mapping out the shortest route to drive our cars or ride a bicycle to some destination, choosing a product to buy from a store and budgeting our living expenses within the constraints of our limited income. In fact, not all the problems we solve in our daily lives involve only calculations. Some problems involve logical thinking and calculation such as finding the most efficient (in terms of driving time and fuel consumption) route from Georgetown, Ontario, to downtown Toronto. Others involve calculating quantities of certain resources such as money, electricity, gas, fuel and water, given certain conditions. Through observations and thinking, you may realize that our daily problem-solving skills and principles are useful when we begin to study formal problem-solving in mathematics.

In this course, a problem is a request to do something according to laid down conditions or constraints. In this case, we know exactly the problems we have to solve and the conditions or constraints under which we have to solve them. This is similar to real-life situations where we have to solve a particular problem by
identifying all conditions our solutions should satisfy. As well, note that problems can take many forms. Some problems require a simple application of skills in number operations; whereas others require application of your skills to familiar or unfamiliar situations.

Business organizations face a number of problems from time to time. These may include how to control inventories, reduce the incidence of shoplifting, determine the price of a merchandise or service, minimize operating costs, provide excellent customer services, and increase worker productivity, sales or profit. These problems may be solved using a variety of skills and knowledge from mathematics, economics, psychology, or logic. And most of these problems may also involve numerical or non-numerical information or both. Since our aim is to make mathematics as easily accessible as any other discipline, we cannot ignore non-numerical information in problem-solving activities.

9.2 Problem-solving Phases

Four phases of problem-solving in this section were taken from George Polya’s book, “How to Solve It”. We have summarized the four phases below:

1. **Understand the problem.** Certainly, you can not solve a problem unless you understand it. In particular, read and identify the information given, the unknown information, and the conditions your solutions should satisfy.

2. **Plan how to solve it.** You should plan how to solve the problem in order to satisfy the given conditions. You should use your previous experience or skills to plan your strategies for solving the problem. In some cases, try to draw diagrams or a picture of the problem. Visualizing the problem in this way, may help in the decision about how to solve it. However, we have suggested a few strategies in this module for solving problems. You should read them carefully, including the illustrated examples.
3. **Try it.** After deciding on a strategy, you should try it, see if it solves the problem satisfies the given conditions.

4. **Look back.** Having solved the problem is not the end of it. Go back to the original problem and make sure that you have satisfied the all the conditions and answered all the questions asked. Also, ask yourself if there are other ways the problem could be solved, and whether your solution is reasonable. Some problems may be solved in many different ways, whereas others may be solved in a few ways.

**9.3 Problem-Solving Strategies**

In this section, we describe and give illustrations of a few strategies for solving mathematics problems. Sometimes two or more strategies have to be combined in order to solve a problem.

**a) Work Backwards.** Here you work from the end to the beginning. It is a common problem in business particularly in auditing cash, hours worked, stock of finished products and raw materials.

**Example 1**

A small retailer had stock of 150 packs of children socks at the end of March 31. During that period, the retailer sold 200 packs of socks and at the same time she received a supply of 300 packs of socks. How many packs of socks did she start with?

**Solution**

We are supposed to find the beginning packs of socks. We are given some information to work with. We can set up a table as follows.

| End with… | 150 |
| Add……… | 200 |
| Subtract…. | 300 |

Start ?

\[150 + 200 - 300 = 50\]

The original number of packs of socks must be 50.

We assume that every stock received has been accounted for and that there are no spoiled, defective, or returned stock.
Let’s check the reasonableness of our answer.
Start with \( X \) (We put \( X \) to hold the place for the packs from the beginning)
Subtract…200 (the packs sold)
Add………300 (the supply received)
End pack...150 (given in the problem)

We see that \( X - 200 + 300 = 150 \)
\[ X + 100 = 150 \]
\[ X = 50 \]
Yes, the beginning packs must be 50

**Example 2**
Fidelia bought some items from Zoom Dollar Store. She paid $7.26, including GST (6%) and PST (8%). How much PST did she pay on the items?

**Solution**
Cost of item \( N \) (\( N \) is holding the place for cost of the items without the taxes)
Add: GST (6%)……
PST (8%)….
Total $7.20 (This is given in the problem)

To work backward, we have to deduct both the GST and PST from the total cost. So, $7.20 \times (6\% + 8\%) = $7.20 \times 14\% \text{ of the cost. Since the } 14\% \text{ was multiplied by the cost of the item and added to it to arrive at } $7.20, \text{ we can take the cost as one whole and adding the taxes will make it } $1.14. \text{ To get the original cost of the items, } $7.20 \div 1.14 = 6.32$
Calculate the PST, $6.32 \times 0.08 = $0.51
Let’s check the reasonableness of our solution.
Original cost of items \( $6.32 \)
Add: PST (0.08 \times 6.32)……… 0.50
GST (0.06 \times 6.32)……….. 0.38
Total cost \( $7.20 \)

**Alternatively**, with the aid of little algebra, we can easily solve this problem.
Let the original price of the items be \( N \). Then we know that this relationship is true:
\[ N + 0.14 (N) = $7.20 \]
\[ N + 0.14N = 7.20 \]
\[ 1.14N = 7.20 \text{ (Divide each side by } 1.14) \]
\[ N = 6.32 \]
Checking, \[ 6.32 + 0.14 (6.32) = 7.20 \]
\[ 6.32 + 0.88 = 7.20 \]
\[ 7.20 = 7.20 \]
With the power of algebra, we can solve complex problems.
b) **Look for a pattern**

It has been said that mathematics is the study of patterns. To solve some problems, we have to see if it has definitive patterns. Then we use the generalized pattern to find a solution to the problem. Do not forget that patterns lend themselves to algebraic methods than they do to arithmetic methods. Once a pattern is identified, translate it into algebraic statements or equations and solve it.

**Example 2**

A sales person notices that she can increase the sales of a new cordless mower over 12 months. On the first month, she sells only one mower, on the second month 5, on the third month 14 and on the fourth month 30.

a) If this trend continues, how many mowers can she sell on the fifth month?
b) If this trend continues, on what month can she sell 285 mowers?

**Solution**

a) After reading the problem, you may have noticed that there is an increasing pattern in the sale of the new mower. We may organize a table like the one below to analyze the pattern:

<table>
<thead>
<tr>
<th>Month</th>
<th>Sales (Quantity)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>?</td>
</tr>
</tbody>
</table>

The difference between the first month sales and the second month sales is 4, which is a perfect square ($2^2 = 4$). Similarly, the difference between the third month sales and fourth month is 16, another perfect square ($4^2 = 16$). According to the pattern, the difference between the fourth month and fifth day sales must also be a perfect square, $5^2 = 25$. Therefore, the fifth day sales must be 55 mowers (30 + 25).

b) Following the pattern discovered in (a), you will see that the sales person will sell 285 mowers on the ninth day.

Is this solution reasonable? Certainly, it is reasonable since it fits perfectly into the pattern we have discovered. In a more advanced way, we could work out a general formula to help us solve the problem. You should review the module on introduction to algebra about how patterns can be translated into formulas or equations. We could have translated the patterns into an equation and solve the resulting equation. However, while this is not necessary for our purpose at the moment, you should try to solve the above problem using algebra as part of your practice.
c) Solve a Simple Version of the Problem

For some difficult problems, the best way to solve them is to solve a simple version of the problems and use the procedure to solve the difficult problems.

**Example 3**

Suppose we have a room with 8 rows of desks and 8 desks in each row. Right now, there is one empty desk in the last desk in the 8th row. Each of the other desks is occupied. Find the minimum number of moves that would have to be made in order to make the first desk in the first row empty, assuming that a desk can have at most one person sitting there, and that nobody can stand. They can only move one seat to the left, right or up, down (not diagonally).

**Solution:** This is a very hard problem with 8 rows and 8 seats. Consider first the case with 2 rows and 2 seats in each as shown below:

<table>
<thead>
<tr>
<th>x</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

In this case, the person in the back moves one to the right and the person in front moves back one (for example) and presto, it takes 2 moves.

Next, consider the 3 by 3 case as shown below:

<table>
<thead>
<tr>
<th>x</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

Stage 1:
Think of the back right corner 2 by 2 square as you would have in the first example. It takes two moves to make the middle square of the 3x3 empty. Then this gives you

<table>
<thead>
<tr>
<th>x</th>
<th>x</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

Stage 2: If we look at the front left 2x2 square and it takes 2 moves, for a total of 4 moves.

One more step should be enough for us to figure out what is happening. Try a 4x4 square as shown below:

<table>
<thead>
<tr>
<th>x</th>
<th>x</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

Stage one: look at the back right 3x3 corner, which we know takes 4 moves, but stage two leaves us with another 2x2 square, so the total is 4+2 = 6.

From this, we can deduce that the number of moves for an 8x8 is 6+2+2+2=12

The general formula for an nxn square would be 2(n-1) moves.
d) Organized Table or list

For some problems, we will have to organize a table or list, and then draw information from the table or list to work out a solution to the problem.

Example 4

Thomas and Smith were enrolled in different training programs. Because Tyrone was junior he was given initial allowance of $600, while Smith was given $1550. However each was given an additional weekly allowance of $100. Tyrone saved all his allowance, but Smith spent all of his allowance plus $90 each week. After how many weeks will they each have the same amount of money?

Solution

The problem we are supposed to solve is this: After how many weeks will they have the same amount of money? According to the given information, Smith spent the allowance of $100 plus $90, a total of $190. We solve this problem by drawing a table and use the information in the table to answer the question.

<table>
<thead>
<tr>
<th>Week</th>
<th>Thomas</th>
<th>Smith</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial</td>
<td>$600</td>
<td>$1550</td>
<td>The start amount</td>
</tr>
<tr>
<td>Week 1: Allow.</td>
<td>+100</td>
<td>+100</td>
<td>1st week allowance</td>
</tr>
<tr>
<td>Spend</td>
<td>0</td>
<td>-190</td>
<td></td>
</tr>
<tr>
<td>Balance</td>
<td>700</td>
<td>1460</td>
<td></td>
</tr>
<tr>
<td>Week 2: Start</td>
<td>700</td>
<td>1460</td>
<td>Start amount</td>
</tr>
<tr>
<td>Allow.</td>
<td>+100</td>
<td>+100</td>
<td></td>
</tr>
<tr>
<td>Spend</td>
<td>0</td>
<td>-190</td>
<td></td>
</tr>
<tr>
<td>Balance</td>
<td>800</td>
<td>1370</td>
<td></td>
</tr>
<tr>
<td>Week 3: Start</td>
<td>800</td>
<td>1370</td>
<td>Start amount</td>
</tr>
<tr>
<td>Allow</td>
<td>+100</td>
<td>+100</td>
<td></td>
</tr>
<tr>
<td>Spend</td>
<td>0</td>
<td>-190</td>
<td></td>
</tr>
<tr>
<td>Balance</td>
<td>900</td>
<td>1280</td>
<td></td>
</tr>
<tr>
<td>Week 4: Start</td>
<td>900</td>
<td>1280</td>
<td>Start amount</td>
</tr>
<tr>
<td>Allow</td>
<td>+100</td>
<td>+100</td>
<td></td>
</tr>
<tr>
<td>Spend</td>
<td>0</td>
<td>-190</td>
<td></td>
</tr>
<tr>
<td>Balance</td>
<td>1000</td>
<td>1190</td>
<td></td>
</tr>
<tr>
<td>Week 5: Start</td>
<td>1000</td>
<td>1190</td>
<td>Start amount</td>
</tr>
<tr>
<td>Allow</td>
<td>+100</td>
<td>+100</td>
<td></td>
</tr>
<tr>
<td>Spend</td>
<td>0</td>
<td>190</td>
<td></td>
</tr>
<tr>
<td>Balance</td>
<td>1100</td>
<td>1100</td>
<td>After the fifth week</td>
</tr>
</tbody>
</table>
From the table, we see that after the fifth week Thomas had $1,100 and Smith also had $1,100. You may see that using a table to organize the information to solve the problem helps us to solve other problems. For example, how much did each have at the end of the fifth week? With how much did Smith start the third week with? Indeed, as an organizing device a table ensures that no information is left out.

e) Guess and check
In practice, the solution to a problem may be worked out through a process of guessing and checking or trail and error. This is the simplest and most natural of all problem-solving strategies. The use of this strategy requires that one make educated guesses at the solution of a problem and constantly checks the guesses against conditions of the problem until the conditions are satisfied or the right answer obtained.

Example 5
An elevator is rated for a maximum load of 1000kg. Estimate the maximum number of passengers allowed if the elevator is to be used by
a) Adults with an average mass of 87 kg.
b) Children with an average mass of 52kg.

Solution
From the problem, we should understand that this relationship is the key:
(# of adults) × (87kg) + (# of children) × (52kg) = 1000kg.

We have to make sure that the weights of adults and children do not exceed 1000kg. Nevertheless, we can not have a total weight that is far too low than 1000kg. We can try a range of numbers less than ten, since if we take ten as the number of adults and ten as the number of children, we will far exceed 1000kg.

Let us try 8; 8 adults and 8 children.

87 (8) + 52(8) < 1000
696 + 416 > 1000
1,112 > 1000

Obviously, the weight of 8 adults and that of 8 children far exceeds the maximum weight by 112. Let us try 7 adults and 9 children.

87(7) + 52 (9) < 1000
609 + 468 < 1000
1,077 > 1000
Again, this is greater than 1000kg but it is better than the previous one.
Let us try 7 adults and 7 children
\[
87 \times 7 + 52 \times 7 \geq 1000
\]
\[
609 + 364 \geq 1000
\]
\[
973 < 1000
\]
Therefore, 7 adults and 7 children should be sufficient to use the elevator, giving a combined weight of 973kg. This seems a better number, since it is less than 1000kg by only 27kg. Perhaps some of the adults or children could weigh more than we have anticipated. In that case, the 27kg is our margin of safety. Some people may suggest that we can add one more child to the weight, bringing it to 1025kg. They may argue that exceed the weight by 25kg will not do any harm to the elevator. Nevertheless, we feel that for the sake of safety we should not exceed the maximum weight.

You should note that we have discussed only a few strategies for solving mathematics problems. The student should not discard other strategies that she/he knows or comes across that may help in solving mathematics problems.

9.4 Review, Exercises and Assignments

1. Ram charges 18 cents per minute for long distance calls. Teleco, on the other hand, totals your phone usage every month and rounds the number of minutes up to the nearest 15 minutes. It then charges $7.90 per hour of phone usage, dividing this charge into 15 minute segments if you used less than the full hour. If your office makes 5 hours 3 minutes worth of calls this month, which of the two phone companies will charge the lower amount?

2. You are preparing to tile the floor of a rectangular office room that has dimensions \( \frac{15\frac{1}{2}}{2} \) feet by \( \frac{18\frac{1}{2}}{2} \) feet. The tiles you plan to use are square, measuring 12 inches on each side and are sold in boxes that contain enough tiles to cover 25 square feet. How many boxes of tiles must you order to complete the job, if you have to allow 15 more tiles for scratches, chips, and other spoilages. (note: 12 inches= 1 foot)
3) Moville Transit posted the following schedule of fares in its recent brochure.

<table>
<thead>
<tr>
<th>Moville Transit Company</th>
<th>Schedule of fares &amp; Rates For Zinton</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Adult</strong></td>
<td><strong>Student</strong></td>
</tr>
<tr>
<td>Single Ride</td>
<td>$5.70</td>
</tr>
<tr>
<td>Day Pass</td>
<td>$11.40</td>
</tr>
<tr>
<td>2 Ride</td>
<td>$11.40</td>
</tr>
<tr>
<td>10 Ride</td>
<td>$52.25</td>
</tr>
<tr>
<td>Monthly Pass</td>
<td>$186.00</td>
</tr>
<tr>
<td>10 Ride</td>
<td>$47.00</td>
</tr>
<tr>
<td>Monthly Pass</td>
<td>$140.00</td>
</tr>
</tbody>
</table>

* Group must not exceed more than 6 students and it is for a single day use only.

a) What is the cost per ride for a student who buys the 10-ride ticket?

b) Tony lives in Moville but goes to the University of Zinton. Tony uses the Moville Transit to commute to classes three times a week. Is it cheaper for Tony to buy the monthly pass or the 10-ride ticket? Support your answer with appropriate calculations.

c) Mrs. Johnson lives in Moville but works in Downtown Zinton. Mrs. Johnson works 5 days a week and commutes by the Moville Transit. Is it cheaper for Mrs. Johnson to commute to Zinton using 10 ride-tickets or a monthly pass? Support your answer with suitable calculations.

d) Whom does the Moville Transit favour: students, adults, frequent riders, or infrequent riders?

4. A social club sent out a monthly newsletter to each of its 400 members. When the cost of postage increased from 48 cents to 53 cents a letter, the club decided to issue newsletters 10 times a year instead.

a) Calculate the percentage increase in the cost of postage.

b) What is the yearly savings that resulted from the decision to send fewer newsletters?
5. A bus-pass costs $98.00. Without a pass, an adult ride costs $2.75.  
   a) In a month, what is the minimum number of times a person with a bus pass must 
      ride the bus in order to save money?  
   b) How much does a person with a bus pass who uses the bus 20 days in a month 
      save?  

6. At downtown parking garage A, it costs $8.75 to park a car for the first hour and 
   $1.25 for each additional hour. At garage B, it costs $5.50 for the first hour and 
   $2.50 for additional hour.  
   a) What is the difference between the cost of parking a car for 5 hours at garage A 
      and for parking it for the same length of time at garage B?  
   b) Approximately how many hours can one park in any of the garages and pay 
      closest to the same same charge, assuming charges are rounded up?  

7. A student’s grade in a math course is determined by 3 quizzes and 1 final 
   examination. 
   a) If the examination counts two times as each of the quizzes, what fraction of 
      the final grade is determined by the examination?  
   b) What percent does the examination contribute to the final grade?  

8. Edna’s gross pay is $490.00 for her first 40 hours of work in a week. She is then 
   paid time and a half times her regular hourly rate for any additional hours. 
   a) How many hours does she have to work in order to make $673.75?  
   b) What assumption did you make in answering the question in a)?  

9. A stock decreases in value by 40 percent. By what percent must the stock price 
   increase in order to reach its former price?
10. The population of a certain town increases by 50% every 50 years. If the population in 1950 was 810, what year was the population 160?

11. Forty people signed up to take a bus trip to Niagara Falls. The senior citizens among the forty people were offered a 20% discount off the regular price. An amount of $1,710 was collected from the 40 people. How many senior citizens went on the trip, if the regular price was $45.00?

12. Mrs. Jones wanted to make a garden in the shape of a rectangle. She has 80 m of fencing to enclose the garden. She wants the length to be 10 m more than twice the width. What are the dimensions, in metres, for the rectangular garden that will use exactly 80 m of fencing?

13. Fred’s favourite photograph has a length of 6 cm and a width of 4 cm. He wants to have it made into a poster with dimensions that are similar to those of the photograph. He decided that the poster should have a length of 42 cm. How many centimeters wide will the poster be?

14. Mike received 10% raise each month for three consecutive months. What was his salary after the three raises, if his starting salary was $2,000 per month?

15. The annual membership fee at Super Golf Club is $245 and it costs $15 to play a game. As part of a fitness program for its managers, Fasco Enterprises has budgeted $8,015 for one year to pay for memberships and golf games for the managers (there is more than one manager). They play an equal number of games.
   a) How many managers were budgeted for?  b) How many games can they play?
   (Hint: the number of managers and number of games is an exact whole number)
16. Mona plans to install a fence around the perimeter of her backyard. Her backyard is shaped like a square and has an area of 800 square m. The company that she hires charges $4.50 per m for fencing and $112 for the installation fees. How much will be the cost of the fence?

17. On Friday Daniel wrote a cheque for $390.47. The following Monday, he deposited $170.02 into his bank account. On Tuesday, the bank deducted $4.24 and $201.00 from his account for service charge and car insurance respectively. On Thursday, the bank informed him that he had overdrawn his account by $180 and since he had no overdraft arrangement with the bank he has to make deposits to wipe the overdraft. Daniel made no other transactions between Friday and Thursday. What was his account balance before he wrote the cheque on Friday?

18. Here is the first four steps in a number pattern.
   
   Step 1: $3 + 2 = 5$
   Step 2: $6 + 4 = 10$
   Step 3: $9 + 6 = 15$
   Step 4: $12 + 8 = 20$

   a) What is step 20 in this pattern?
   
   b) Which of these is part of the pattern above?
      i) $56 + 28 = 84$
      ii) $75 + 50 = 125$
      iii) $120 + 60 = 180$
      iv) $100 + 50 = 150$
19. A cheque may bounce when the payee has insufficient money or has no special agreement with a bank. One retail store charged 6% penalty on the amount on a cheque and $5 fee for administrative cost. Another retailer charges only a $15 fee for a bounced cheque.

a) When will the total cost of bounced cheques be the same for both stores?
   Use mathematics to justify your answer.

b) What is the total charge by each store for a cheque amount of $289.50? Use mathematics to explain how you determined your answer.
U5- Full Chapter 10 Introduction to Mathematics of Finance

10.1 What is Interest?

Interest is either the amount of money you pay for using somebody’s money or the amount you receive for allowing somebody to use your money for a specific length of time. In the first sense, interest is regarded as a charge or cost; and in the second sense, interest is regarded as income. Interest is always a rate, calculated by dividing the total amount interest by the amount of the loan.

When Banks and other institutions lend out money to people through credit cards (master cards, visa, American Express, etc.), loans, overdrafts, and credit-line they charge them interest. As well, businesses and governments pay money to some people when these people allow them to use their money. For instance, some people buy Canada savings bonds, which is a way the Federal government of Canada borrows money. The Federal government then pays interest to the bondholders. Some people invest their money in fixed deposits such as savings account, and Guarantee Investment Certificate (GIC) for the purpose of earning interest.

Furthermore, businesses that provide utilities such as water, hydro, gas, cable and phone often charge their customers interest for bills that remained unpaid after the due date. This may be justified on the grounds that, any money owed to those utility companies after the due date is, in effect, money lent out to the customers. After all, the customers have used up the service and are required to pay for it. A failure to pay for it after the established due date implies that the utility company has lent out money to the customer, giving the company the right to charge interest. However, the utility companies do not pay any interest to customers who pay their bills earlier than the due date. This has led some people to argue that the utility companies use interest charge as a punitive measure with the intent to make money off the poor. Nevertheless, it should be noted that the utility companies send out bills to their
customers after the services had been consumed. Therefore, when one pays one’s utility bill earlier than the due date it does not give the companies any financial advantages. In the business world, historically, there were two basic types of interest: simple and compound. Simple interest is rarely used today. It is applied for tax purposes, for example, to determine the amount by which an asset (like a care, for example) loses money each year and this does not change, as it is always based on the original value of the item. Compound interest, on the other hand, applies to savings accounts, installment loans, credit cards, and overdue utility bills. The amount of interest paid is based on the current value of the investment or loan. Both types of interest take into account three factors: the principal, the interest rate, and the time period involved. Principal is the amount of money borrowed or invested. Rate is the percent of the principal paid as interest per time period. Time is the number of days, months, or years for which the money is borrowed or invested.

10.2 Another application of Percent : Simple Interest

A sum of money borrowed may be called principal of the loan, and the sum of money paid back (principal + interest) is called the maturity value. Often times, the interest is quoted as a rate of percent, denoted by \( r \). The time for which the money is used is also denoted by \( t \). Suppose you borrowed $6000.00 from your credit line for six months. And the bank charged you $40 a month for interest. The principal of the loan is $6000.00, interest is $240.00 and the amount is $6,240.00 \{\text{principal $6000 + interest of $240(40 x 6)}\}. Since the interest per month is $40, the interest per year is $480 \(40 \times 12\). We can convert the interest amount per year into percent rate: \( \frac{480}{6000} \times \frac{100}{1} = 8\% \). The interest rate for one month= \( 8\% \div 12 = 0.67\% \). The interest payable per month = \( \frac{0.67}{100} = 0.0067 \times 6000 = $40 \).
We can think of an interest charge as how many dollars per $100 borrowed for a time period. For instance, if you borrowed $4000 to buy a car for one year at 5.25% per annum, the interest charge is $5.25 per $100 per year. Since 4000 has 40 hundreds, the interest charge for one year is $5.25 \times 40$ or $210$. For six months, the interest is $5.25 \times 40 \times \frac{1}{2} = $105.00 (Note that $\frac{6}{12} = \frac{1}{2}$, using 6 as a common divisor or factor). For 8 months, the interest is $5.25 \times 40 \times \frac{8}{12} = $140. For three years, the interest is $5.25 \times 40 \times 3 = $630.00. From this example, we should notice that interest charge depends on the principal, percent rate, and the time-period. For this reason, interest = principal x rate x time or, $I = PRT$; where $I = \text{interest}$, $P = \text{principal}$, $R = \text{Rate of interest}$, $T = \text{Time}$ (in years, months, weeks, days, or hours). This formula is called basic simple interest formula.

**Example 1**

Write the yearly interest rates equivalent to the following monthly rates.

a) 2%  
   b) $1 \frac{1}{2}$%  
   c) $\frac{3}{4}$%  
   d) $2 \frac{1}{4}$%  
   e) 1%  
   f) $2 \frac{1}{2}$%

**Solution**

a) $2\% \times 12 = 24\%$  
   c) $\frac{3}{4} \times 12 = 9\%$  
   e) $1 \times 12 = 12\%$

b) $1 \frac{1}{2} \times 12 = 18\%$  
   d) $2 \frac{1}{4} \times 12 = 27\%$  
   f) $2 \frac{1}{2} \times 12 = 30\%$

**Example 2**

Give the monthly rates equivalent to the following yearly rates (Give your answer in fraction).

a) 12%  
   b) 6%  
   c) 18%  
   d) 9%  
   e) 5.45%  
   f) 28% 

**Solution**

a) $\frac{12}{12} = 1\%$  
   b) $\frac{6}{12} = \frac{1}{2}\%$  
   c) $\frac{18}{12} = \frac{3}{2} = 1 \frac{1}{2}\%$

d) $\frac{9}{12} = \frac{3}{4}\%$  
   e) $\frac{5.4}{12} = \frac{17}{6} = 2 \frac{5}{6}\%$  
   f) $\frac{28}{12} = \frac{7}{3} = 2 \frac{1}{3}\%$
Example 3

Express each interest rate as a charge for $100 borrowed for the indicated time.

a) 6% per year       b) 3% per month     c) 5% per month

d) 10% per year      e) $8 3/4 % per month   f) 1% per day

g) 7% per year

Solution

a) $6.00 per $100 for a year
b) $3.00 per $100 for a month
c) $5.00 per $100 for a month
d) $10.00 per $100 for a month
e) $8.75 per $100 for a month
f) $1.00 per $100 for a day
g) $7.00 per 100 for a year

10.3 Find Interest and Maturity Value

The interest payable on a loan can be easily calculated if the principal is perfectly a multiple of 10. Otherwise, we have to calculate the interest using the basic simple interest formula established earlier,

\[ I = PRT \]

The amount of the loan or investment plus interest due by the end of the loan period is called the maturity value. The maturity value can be calculated either by adding the principal and the interest or directly from the relationship between the principal, rate, and time.

Approach 1

If the principal and interest are known, the maturity value is calculated as follows:

\[ \text{Maturity value} = \text{Principal} + \text{Interest} \]

\[ \text{MV} = P + I \]
Approach 2

Maturity value can be calculated directly, when principal, rate, and time are known.
\[ MV = P + PRT \]
\[ MV = P \times (1 + RT) \]
We factor out the \( P \); This is an application of the distributive property. This formula means add 1 to the product of the rate and time and multiply the sum by the principal. Remember the order of operation that says we have to work inside a bracket first.

The two examples illustrated below show how to apply this formula in calculating interest. The basic simple interest formula can be applied regardless of the period for which a loan is lent or borrowed - monthly, weekly, daily or hourly.

Example 1
Smith borrowed $4,560 for 1 year at 4% per year. How much interest will he pay for \( 1 \frac{1}{2} \) years?

Solution
\[ P = 4,560 \quad R = 4\% \quad T = 1 \frac{1}{2} \]
\[ I = PRT = 4,560 \times 0.04 \times 1.5 = 273.50 \]
The interest payable for \( 1 \frac{1}{2} \) years is $273.50.

Example 2
Joan took a car loan of $7550 at a monthly rate of 2% per month from Continental bank.

a) What is the interest chargeable for a year?
b) What is the maturity value for one year?

Solution
a) The interest rate is monthly. We should either convert the monthly rate to yearly rate by multiplying the one month’s interest by 12.
\[ P = 7550, \quad R = 2 \times 12 = 24\%, \quad \text{time} = 1 \]
I = 7550 x 0.24 x 1 = $1,812.00. The interest is $1,812.00.

Or I = 7550 x 0.02 x 12 = $1,812.00

b) Maturity value = Principal + Interest = $7550 + $1,812 = $9,362.00

or MV = P (1 + RT)

MV = 7550 \{1 + (0.24 \times 1)}

MV = 7550 \{1.24\}

MV = $9,362.00

**Example 3**

A used car is purchased for $15,200, and a down payment of $1,200 is made. The balance is financed for 3 years at an annual interest rate of 5%.

a) Find the amount financed.

b) Find the monthly payment.

**Solution**

a) The car costs ........$15,200

Deduct down payment 1,200

Amount to finance $14,000

b) To calculate the monthly payment, we have to find the interest first.

Interest = principal x annual rate of interest x time (years)

= 14,000 x 0.05 x 3

= 2,100.00

Monthly payment = maturity value ÷ length of loan in months

= (14,000 + 2100) ÷ 36 (12 x3)

= 16,100 ÷ 36 = $447.22

The monthly payment is $447.22
10.4 Finding Principal, Rate, or Time

Some times, we may have a situation where we have to find the principal, rate or time, given the interest. When this occurs we have to rearrange the basic formula, \( I = PRT \). If principal is the unknown quantity, then it is the subject of the formula. The same thing applies to the rate of interest and time. To rearrange the basic formula, we have to use the principles of equality we learnt in section three. Alternatively, we may use just substitute the values we have into the formula and find the unknown.

**Example 1**

Interest on a certain sum of money for 1 month is $50 at the rate of 2% per month. How much was the principal?

**Solution**

We simply plug the values into the formula and find the principal

\[
I = PRT
\]

\[
12 = P \times 0.02 \times 1 = 0.02P
\]

\[
12 = 0.02P
\]

We have to solve the equation for \( P \). Divide each side of the equation by 0.02.

\[
\frac{12}{0.02} = P
\]

\[
P = 600
\]

The principal is $600.

The alternative is to rearrange the basic formula and make \( P \) its subject.

\[
I = PRT
\]

To make \( P \) the subject of the formula we have to isolate it by dividing each side by \( RT \). That is, since \( P \) is multiplying \( R \) and \( T \) together we can only isolate \( P \) by division. You should remember that division is the inverse of multiplication.

\[
\frac{I}{RT} = P
\]

We now substitute the values into the rearranged formula.

\[
\frac{12}{1 \times 0.02} = P
\]

\[
P = 600
\]

So the formula for finding the principal given the interest is \( \frac{I}{RT} \).
Using the same technique, the formula for \( R = \frac{I}{PT} \) and \( T = \frac{I}{PR} \)

### 10.5 Finance Charge and Credit Card Bill

Using a credit card for purchases is similar to receiving a loan from the credit card company. Therefore, credit card companies charge their customers annual fee and interest on purchases or cash advance made using credit cards. These charges are called **finance charges**.

The finance charge on a credit card is calculated using the simple interest formula. Since credit card companies usually send out their monthly bills, the interest rates on credit purchases are monthly rates. However, when interest rates are expressed in annual rate, we have to convert it into a monthly rate by dividing it by 12.

**Example 1**

Tory uses a credit card that charges 23\% per annum. Find his finance charge when her unpaid balance in one month is $1200 (Correct to two decimal places).

**Solution**

Monthly interest rate = \( 23\% ÷ 12 = 1.92 \)

Interest charge = Principal x monthly interest rate x time (in month)

\[
= 1200 \times 0.0192 \times 1
\]

\[
= $23.04
\]

Tory’s finance charge is $23.04.

**Example 2**

An unpaid balance on a credit card is $7,200 and the monthly interest card is $52.45.

a) What is the monthly interest rate?

b) What is the annual interest rate on the credit card?
Solution

a) We state the formula and substitute the values in their appropriate places. We use \( y \) to represent the unknown, the monthly interest rate.

Interest charge = Principal \( \times \) monthly interest rate \( \times \) time (in month)

\[
52.45 = 7,200 \times y \times 1
\]

\[
52.45 = 7,200y \quad \text{After simplifying by multiplication.}
\]

\[
\frac{52.45}{7,200} = \frac{7,200y}{7,200} \quad \text{Divide each side by 7,200 to isolate } y.
\]

\[
\frac{52.45}{7,200} = y
\]

\[0.73\% = y \]

The monthly rate of interest is 0.73%.

b) The annual rate \( = 0.73 \times 12 = 8.76\% \)

10.6 Review, Exercises and Assignments

1. Explain the meaning of each of the following terms.
   a) Principal  b) Time  c) Interest  d) Rate

2. Interest may be regarded as either expense or revenue. True or false?
   Explain your answer.

3. Express each percent as a fraction in lowest term.
   a) 40\%  b) 75\%  c) 0.50\%  d) 20\%

4. Express each percent in decimal notation.
   a) 23\%  b) 250\%  c) 0.003\%  d) 0.01\%  e) 4.5\%

5. Express each number as a percent.
   a) 0.45  b) 1.25  c) 0.007  d) 0.04
6. If $120 interest is paid on $2000 borrowed for three months, what is the interest rate?

7. a) What percent is equivalent to \( \frac{24}{150} \)?
   
b) What decimal is equivalent to 2.5%?

8. a) Write 0.034 as percent?
   
   b) Write 3.05% as a decimal?

9. An amount of $2,250 was borrowed at 6% per annum.
   
a) What is the interest rate per month?
   
b) What is the interest charge for $2,250 per annum?
   
c) What is the interest charge for $2,250 for 5 years?

10 a) Which is greater \( \frac{1}{40} \) or \( \frac{1}{4} \) %?
   
   b) Which is smaller \( \frac{1}{2} \) % or 0.04 ?

11. a) Simple interest may be viewed as interest charged per 100 dollars borrowed.
    
    How much interest will you pay for $5000 borrowed six months at 5.5% per years?
    
    b) What yearly interest is equivalent to 3% per month?

12. Find the simple interest earned on each of the following.
   
a) $1500 deposited for 6 months at 5% per year.
   
b) $5000 loaned for 2 years at 7.5% a year.
13. Calculate the simple interest on $1800 at 6% for
   a) 5 years       b) 2months     c) 40days
14. a) If 25% of a number is $300, what is the number?
   b) Express $\frac{4}{5}$ as a percent.

15. a) $V=IR$, rearrange the equation so that $R$ is the subject.
   b) Solve for $K$, when $P = C + NK$
   c) Solve for $P$, $TA = P + I$

16. June borrowed $4,275 for one year and paid $256.50 in interest charges.
    What was the rate of interest?

17. Musah borrowed $3,000 at 5% per annum. The interest charge was $75. How long was the loan?

18. How many months at 5% simple interest will $3,360 amount to $3,402?

19. Joe operates a pizza hut. He borrowed $25,000 from his bank for six months. At the end of that period, he repaid the principal and the interest for $2567.50. What interest rate was he is paying?

20. A hydro company charges 2% per month when the due for a bill passes. Mr. Winston’s family hydro bill was $215 and he was 45 days late for paying the bill.
    a) How much interest will the hydro company charge him for interest?
    b) What was the total amount he must pay to the hydro company?
21. The simple interest formula is \( I = PRT \) or \( I = P \times R \times T \). Rearrange the formula to find \( T \).

22. What is the interest payable when $30,000 was borrowed at 6.10% for 320 days?

23. Some utility providers charge interest for overdue bills. Suggest two reasons for such practice.

24. \( T \) stands for the number of years or months. \( P \) stands for principal of a loan, and \( R \) stands for rate of interest. Explain the meaning of the formula \( I = PRT \).

25. Jennifer is comparing simple interest rates. Which of the following is the lowest interest rate? How do you know?
   a) 0.05% per day   b) 0.35% per week   c) 1.6% per month   d) 18.1% per year.

26. Convert into years in fractional form.
   a) 8 months   b) 40 months   c) 28 months   d) 18 months

27. Find the maturity value that a borrower will pay back on a loan of $14,000 at \( 12 \frac{1}{2} \% \) annual simple interest for 3 years.

28. Kelvin decided to establish a shoe repair shop. He invested $5,500 for 45 months at \( 10 \frac{1}{2} \% \) simple interest per year.
   a) How much interest did he earn?
   b) How much money did he receive at the end of the 45 months?

29. Mike borrowed $25,000 for four years to purchase a car. The simple interest loan has a rate of 8.5% per year. What is the maturity value of the loan?
30. Progressive Investors Inc agreed to lend money to Joe at special interest rate of 8% per year. The condition is that Joe will borrow enough so that he would pay $500 interest over a two-year period. What was the minimum amount he could borrow?

31. Tasha needed money for university tuition. She borrowed $6,000 at 10% simple interest per year. If she paid $400 interest, how long was she given the loan?

32. Ruth needed money to buy lawn equipment. She borrowed $500 from her credit line for seven months and paid $53.96 interest. What was the rate of interest?

33. Ron received $1,440 on an investment savings account of $12,000 at 6% annual simple interest rate. How long was the money invested?

34. You took a loan of $5,000 at 6% per annum. What is the monthly interest payment?

35. A credit card company charges a customer 1.25% per month on any unpaid balance on its credit cards. What is the finance charge when a customer has an unpaid balance of $1,100?

36. In how many months will $2500.00 earn $182.29 interest at 12.5%?
37. Financial Special is a credit card company that charges 1.45% on unpaid balance on its credit cards and an annual fee of $12.00. Saving super is also a credit card company that charges 1.75% per month on unpaid balance but does not charge any annual fees.
   a) What is the difference between the finance charges these two companies assess on an unpaid balance of $2,100?
   b) Which of these companies gives favourable terms to consumers?
   c) Apart from interest rate and annual fee, what other factors will you consider in choosing a credit card?

38. Mountain Top Homes obtained a preconstruction loan of $500,000 for 8 months at an annual interest rate of 9.5%. What is the simple interest due on the loan?

39. What is the annual finance charge on unpaid balance of $1,438.35 on a credit card that charges 1.15% per month and an annual fee of $35.00?

40. Transmission Supreme borrowed $125,000 for construction of a new premises and financed the full amount at 8% annual simple interest for 2 years. The simple interest on the loan is $20,000. Find the monthly payment.

41. Willy borrowed $10,000 loan at an 8.2% annual simple interest rate for 9 months. Find the maturity value of the loan.
42. Fred has borrowed $40,000 to buy a car. He wishes to repay the loan over three years. Which of the following borrowing options requires her to pay the least amount of simple interest?
   a) 7% per annum for the entire period.
   b) 8% per annum after a 6 month of free interest.
   c) 11% per annum after a 12 month interest free period
   d) 14% per annum after 18 month interest free period.

43. Erica bought a computer on credit. She agreed to pay $500 for deposit and $80 per month. To calculate the total amount he owes, what information does Erica need?

44. Frank has a credit card with an interest rate of 0.50% per day and no interest free period. Frank used the credit card to pay for car repairs costing $580. He paid the credit card amount 16 days later. What is the total amount (including interest) that he paid for the repairs?

10.7 Another Application of Percent: Compound Interest

We start this section by studying carefully the table below. Make sure that you understand how the numbers in the columns and rows were obtained. Ask yourself some questions as you work through the table.

<table>
<thead>
<tr>
<th>Year</th>
<th>Principal</th>
<th>Interest Charge</th>
<th>Maturity Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st year</td>
<td>$5,000.00</td>
<td>$250.00</td>
<td>$5250.00</td>
</tr>
<tr>
<td>2nd year</td>
<td>5,250.00</td>
<td>262.50</td>
<td>5,512.00</td>
</tr>
<tr>
<td>3rd year</td>
<td>5,512.50</td>
<td>275.63</td>
<td>5,788.13</td>
</tr>
<tr>
<td>4th year</td>
<td>5,788.13</td>
<td>289.41</td>
<td>6,077.54</td>
</tr>
<tr>
<td>5th year</td>
<td>6,077.54</td>
<td>303.88</td>
<td>6,381.42</td>
</tr>
</tbody>
</table>
What did you learn from the table? How do you compare the interest calculated to simple interest? Do you see any differences or similarities?

We may note the following patterns about the above table.

1. The amount of interest increases every year. For instance, the first year interest is $250.00 compared to the third year interest of $275.63. This implies that the investor or lender earns more interest than the previous time.

2. The principal at the beginning of each year increases too. For example, the second year’s principal is $5250.00 compared to the fifth year’s principal of $6,077.54. This is because the principal of the beginning year increases by the interest from the previous year.

3. For the first year, we could have obtained the amount by multiplying 5000 by 1.05 instead of calculating the interest separately and adding it to the principal. We could have done the same thing for the second, third, and fifth year. We could obtain the amount for the fifth year by:

\[ 5000 \times 1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 = 5000(1.05)^5 = 6,381.41 \]

The above table is an example of compound interest. Instead of giving back the interest to the investor, it is added directly to the money loaned or deposited. The new balance or principal is always higher than the amount of the preceding principal. This allows the investor or the lender to earn more interest than before.

Let us calculate the interest using the simple interest method.

\[ I = PRT = 5,000 \times 0.05 \times 5 = 1,250.00 \]

The maturity value after 5 years = 5,000 + 1,250 = $6,250

Compare this to the compound interest

The maturity value after 5 years (from the table) =$6,700.45

Deduct the principal = - 5,000.00

Compound interest after 5 years = $1,700.45
Compound interest earns \$450 = (\$1,700 - 1,250) more interest than the simple interest.

10.8 Comparing Compound Interest and Simple Interest
We summarize the differences between compound interest and simple interest in the following table below.

<table>
<thead>
<tr>
<th>Compound Interest</th>
<th>Simple Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. A complex way of looking at interest.</td>
<td>1. A basic way of looking at interest</td>
</tr>
<tr>
<td>2. A more involved calculation</td>
<td>2. A simple calculation</td>
</tr>
<tr>
<td>3. Interest from the previous period also earns interest in the succeeding period.</td>
<td>3. Interest does not earn additional interest</td>
</tr>
<tr>
<td>4. Interest increases provided no withdrawals are made.</td>
<td>4. Interest is a constant amount.</td>
</tr>
<tr>
<td>5. Earn more interest than simple interest.</td>
<td>5. Earn less interest than compound interest.</td>
</tr>
<tr>
<td>6. Examples of interest calculated using compound interest method are credit cards (visa, master card, American Express, etc.)</td>
<td>6. Examples are car loan, personal loan, credit line, etc.</td>
</tr>
</tbody>
</table>

utility bills overdue, etc.
10.9 Deriving Compound Interest Formula

Remember that we showed in the table above that to calculate the amount payable after five years, we could simply calculate it with the help of the formula,

$$5,000(1 + 0.05)^5$$

In general terms, we can use letters to construct the formula as follows.

$$MV = P (1+ \frac{r}{n})^{nt}$$

where

- **MV** = Future value or the amount accumulated after the interest is compounded.
- **P** = The present value or the principal amount of deposit or investment.
- **r** = The rate of interest. If interest is being compounded more than once, **r** is divided by the number of times the interest is compounded.
- **n** = The number of years over which interest is collected or charged (compounded).
- **t** = The number of years for which the principal is invested or borrowed.

10.10 Period of Compounding

It is very important to understand the period of compounding, or the internals when interest is calculated.

**Example 1 (Compounded yearly)**

A principal of $10,000 was interested for 4 $\frac{1}{2}$ years compound interest paying 5% compounded yearly. How much will be earned in 4 $\frac{1}{2}$ years?

**Solution**

Since interest is compounded annually, n= 4.5, r = 0.05

$$FV =10,000 (1+ 0.05)^{4.5} = $12,455.23$$

The amount is $12,455.23
Example 2 (Compounded Semiannually)

A principal of $10,000 was invested for $4\frac{1}{2}$ years compound interest paying 5% compounded semi-annually. How much will be earned in $4\frac{1}{2}$ years?

**Solution**

\[ r = \frac{5}{2} = 2.5\% \quad \text{or} \quad 2.5\% \quad \text{n} = 4.5 \times 2 = 9 \]

\[ FV = 10,000(1 + 0.025)^9 = 10,000 (1.025)^9 = $12,488.63 \]

The amount is $12,488.63.

Example 3 (Compounded Quarterly)

A principal of $10,000 was invested for $4\frac{1}{2}$ years compound interest paying 5% compounded quarterly. How much will be earned in $4\frac{1}{2}$ years?

**Solution**

\[ r = \frac{5}{4} = 1.25\% \quad \text{or} \quad 1.25\% \quad \text{n} = 4.5 \times 4 = 18 \]

\[ PV = 5,000 \times \left(1 + 0.0125\right)^{18} = $12,505.77 \]

The amount is $12,505.77.

Example 4 (Compounded monthly)

A principal of $10,000 was invested for $4\frac{1}{2}$ years compound interest paying 5% compounded monthly. a) How much interest will be earned in $4\frac{1}{2}$ years? b) How much will be the maturity value of the investment after that time?

**Solution**

a) \[ r = \frac{5}{12} = 0.42\% \quad \text{n} = 4.5 \times 12 = 54 \]

\[ MV = 10,000 \times (1 + 0.0042)^{54} = $12,539.83 \]

The compound interest is $2,539.83 ($12,539.83 − 10,000).

b) The maturity value is $12,539.83
Example 5 (Compounded daily)

A principal of $10,000 was invested for $4 \frac{1}{2}$ years compound interest paying 5% compounded daily. How much will be earned in $4 \frac{1}{2}$ years?

**Solution**

\[ r = \frac{5}{365} = 0.01\%, \text{ nt } = 4.5 \times 365 = 1642.50 \]

\[ MV = 10,000 \times (1 + 0.001)^{1642.50} = \$51,793.41 \]

The amount is $51,793.41.

The above examples show that the more the interest is compounded the more the amount payable at the end of the period grows.

We summarize below a procedure for calculating the period interest rate and the interest period. This is the difficult part of compound interest for most students.

**10.11 To find period interest rate**

Divide the annual interest rate by the number of interest period per year.

Period interest rate = Annual interest ÷ number of interest period per year

Interest period = Number of years × number of interest period per year

**10.12 Comparing Compound interest Rate and Simple Interest Rate**

A simple interest rate can be converted to an equivalent compound interest rate. This helps to compare simple interest and compound interest in a more meaningful way. The annual equivalent rate (AER), effective annual interest rate or effective rate takes into consideration the effects of compounding and inflation. When identifying the rate of earning on investments, the effective rate may be referred to as **annual percentage yield (APY)**. However, it is called **annual percentage rate (APR)** when it is payable on a loan. The federal and provincial regulations in Canada requires that companies must disclose the annual percentage rate (APR) charged for a loan, mortgage or overdraft. In some cases, the APR differs from the effective interest rate where it includes additional charges such as insurance, processing fee, document and
preparation fees. But in either case, the APR or the effective interest rate is higher than the actual rate of interest.

Effective interest rate (EAR) is calculated as follows: Compound interest for the period ÷ principal.

Alternatively, we can calculate the effective interest rate by using the formula,

\[(1 + \frac{i}{n})^n - 1;\]

where \(i\) is the stated rate of interest and \(n\) is the frequency of compounding or compounding period.

**Example**

A loan of $4,000 was taken for one year at 12% annual compounded quarterly. What is the effective interest rate?

**Solution**

Period interest rate = 12% ÷ 4 = 3% = 0.03

First end-of-period principal = 4000(1+0.03)

= 4,000 (1.03)

=$4,120.00

Second end-of-period principal = 4120 (1.03)

=$4,243.60

Third end-of-period principal = 4243.60(1.03)

=$4,370.91

Fourth end-of-period principal = 4370.91(1.03)

=$4502.04

Compound interest for the period = 4,502.04 – 4,000 =$502.04

Effective interest rate = \(\frac{502.04}{4000}\) = 0.13 or 13%. The effective interest rate is 13%.

Note: this rate of interest is a simple interest that will give us the same interest of $502.04. (0.13 × 4000)
We obtain the same answer using the formula, \((1 + \frac{i}{n})^n - 1\).

\[ i = 12\%, \ n = 4 \text{ (compounded four times a year)} \]

\[ \text{EAR} = (1 + \frac{12}{4})^4 - 1 \]

Work within the bracket first. Don’t forget that division comes first before addition.

\[ \text{EAR} = (1 + 0.03)^4 - 1 = (1.03)^4 - 1 = 1.13 - 1 = 13\% \]

You should note that when compounding occurs once a year, there is no difference between Annual Percentage rate (APR) and effective interest rate or effective annual rate (EAR). However, effective interest rate takes into account the effects of compounding and inflation. Effective interest is important when interest is compounded more than once a year.

Note that if you have an access to an electronic calculator with \(y^x\) function key, you can calculate the numerical value of the compounding factor by following the steps below:

1. Step1. Enter (1+i) using the keypad
2. Step2. Press the \(y^x\) function key
3. Step3. Enter the numerical value of \(n\) using the keypad.
4. Step4. Press = sign to display the result.

The use of an electronic calculator can minimize the drudgery of computation.

However, the student should note that some calculators give approximation to two or three decimal places. Therefore, depending on the type of electronic calculators used, there may be little differences in the final answers to questions. But the differences may be very insignificant.
10.13 Finding Present Value (Principal)

Sometimes it may be necessary to find out how much money should be invested now in order to receive a specific sum of money or future value. For example, if we want the present value of $500, it means how much should be invested at a certain rate of interest and time in order to have $500 at the end of the period. We provide a simple illustration of this concept, using a manual calculation method. There is a present value table that could be used to avoid laborious calculations.

To calculate the present value of an investment, we either have to rearrange the basic compound interest formula or use the formula as it is and solve for P.

Rearranging the Formula we have,

\[ MV = P \left(1 + \frac{r}{n}\right)^n \]

\[ \frac{MV}{(1 + \frac{r}{n})^n} = \frac{P(1 + \frac{r}{n})^n}{(1 + \frac{r}{n})^n} \]

Divide each side of the formula by \( (1 + \frac{r}{n})^n \).

\[ \frac{MV}{(1 + \frac{r}{n})^n} = P \]

**Example 1**

A manufacturer of garage doors needs $100,200 in five years to purchase a set of two new machines to replace its wearing machines. The money market now pays 7% compounded semiannually. How much should be invested now?

**Solution**

MV = $100,200

Annual interest rate = 7%

Time = 5 years
Compounding period (n) = 2 times a year

\[ MV = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[ 100,200 = P \left(1 + \frac{0.07}{2}\right)^{2 \times 5} \]

\[ 100,200 = P \left(1+0.0350\right)^{10} \]

\[ 100,200 = P \left(1.0350\right)^{10} \]

\[ 100,200 = P \left(1.41\right) \quad \text{Divide both sides by 1.41} \]

\[ P = \frac{100,200}{1.41} = $71,063.83 \]

The company has to invest $71,063.83

10.14 Review, Exercises, and Assignments

1. Check to make sure that an investment of $5,600, invested at 7.2% per annum compounded monthly, has a value of $8018.02 after five years.

2. Compare the values of an investment of $5,600 after 12 months if it earns 5.2%
   Compounded, a) annually  b) quarterly  c) monthly  d) daily

3. What is the better way to invest $500 for five years?
   a) at 5% simple interest
   b) at 4.5% interest compounded monthly
   c) at 4.4% interest compounded daily.

4. Fidelia deposited $3150 in a savings account that earned 4.8% interest per year.
   a) How much simple interest would she earn in 5 years?
   b) How much compound interest would she earn in 5 years?
   c) Compare the two methods of calculating interest. Which method earn more interest? Why?
5. Regina sold her old computer for $750 and decided to put the money in a bank account that is paying 10% interest.

a) How many years would it take her to double her investment to $1,500, using simple interest method?

b) How long would it take her to double her investment to $1,500, using the compound interest method?

c) What is the advantage of calculating interest by compounding?

6. Compare the two options below.

   Option 1. Invest $500 at 6% compounded monthly.
   Option 2. Invest $500 at 6% compounded annually.

Which option makes more money? Support your answer with suitable calculations.

7. Find the maturity or future value of an investment of $20,500 if is invested for four years and compounded quarterly at an annual rate of 8%. How much is the compound interest over that period of time the money was invested?
8. Use the internet or other sources to investigate the following information about credit cards.

<table>
<thead>
<tr>
<th>Credit Cards</th>
<th>Where it can be used</th>
<th>User Fees</th>
<th>Annual Interest</th>
<th>Daily Interest Rate</th>
<th>Minimum Payment</th>
<th>Methods of bill payment</th>
<th>Incentives to use</th>
<th>Options for lost card</th>
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a) Which credit card (s) do you think offers the best features for consumers?
b) Which credit card (s) offers the least features for consumers?
c) Which credit card do you prefer and why?

9. Find the interest on a loan of $5,000 for six years at 7.2% annual interest compounded semiannually.

10. Danielson plans to buy a house in four years. He will make a $20,000 down payment on the property. How much should he invest today at 6.5% annual interest compounded quarterly in order to have the required amount in four years?
11. How much money should Oiler Trailer Inc invest today in order to have $18,000 in one year to purchase a forklift truck, if the interest rate is 7% compounded semiannually?

12. If you were offered $500 today or $580 in one year, which one would you accept if the money can be invested at 12% annually compounded monthly?

13. Mark invested $2,500 in a savings account that pays 8% annual interest compounded daily. Find the value of the investment after 9 years.

14. Suppose you have a savings account that earns interest at the rate of 6% per year compounded monthly.
   a) On January 1, you open the account with $500 deposit.
   b) On February 1, you deposit an additional $200 into the account. What is the value of the account after the deposit?
   c) On July 1, you deposit additional $1,000 into the account. What is the value of the account after the deposit?
   d) On November 1, you withdraw $150 from the account. What is value of the account at December 31?

15. To save for her retirement, Hannah deposited $5000 into an account that pays 7% annual interest compounded daily.
   a) What will be the value of the account after ten years?
   b) How much interest will be earned in ten years?
16. A farmer borrowed $25,000 for three years to buy a tractor at the annual interest rate of 7.3% compounded monthly.
   a) What is the compound interest on the loan?
   b) What is the effective interest rate?

17. Explain the following terms.
   a) Simple interest
   b) Compound interest
   c) Maturity value of a loan
   d) Effective interest rate
   e) Present value

18. An investment of $1,000 is made at the beginning of each year for three years. The investment is compounded semiannually at 8%. Find the maturity value and the compound interest at the end of the third year.

19. Francis was offered $15,000 cash or $19,500 to be paid in two years. If money can be invested in today’s money market for 7% annual interest compounded quarterly, which offer should Francis accept?

20. Explain why MV = P + I and MV = P(1 + R) are related.

21. A loan of $15,000 was taken for two years at interest of 8% compounded semiannually.
   a) Calculate the compound interest on the loan.
   b) Calculate the effective interest rate for the loan.
22. A used car was purchased in July 1999 for $11,900. If the car depreciates 13% of its value each year, what is the value of the car, to the nearest hundred dollars, in July 2003?

23. $2,000 is invested in a bank account. The account earns a compound interest at 5% per year.
   a) What is the cash value of the account, to the nearest dollar, at the end of 5 years?
   b) How long will it take the investment to double itself?

24. Mathew’s credit card balance was $1,300 at the end of July. Because Mathew lost his job he couldn’t pay the balance until the end of December. If the credit card company charges 1.5% per month on any unpaid balance, how much did Mathew pay at the end of December?

25.”Compound interest is calculated not only on the original principal but also on the interest already earned”. Explain with an example.

26. What is the maturity value of $10,000 at 10.5% compounded monthly
   a) in five years?   b) in six and half years?   c) in ten years?   d) in 15 years?

27. Explain three factors that distinguish compound interest from simple interest?

28. For a sum of money invested at 8% compounded quarterly for 10 years, state the following.
   a) The number of compounding period
   b) The interest rate for each period
c) The compounding factor in exponential form

d) The numerical value of the compounding factor.

29. For each of the interest rates below, determine the compounding rate.
   a) 5% compounded quarterly.
   b) 75 compounded monthly.
   c) 5.7% compounded semi-annually.
   d) 8% compounded annually.
   e) 10% compounded daily.

30. For the compound interest formula determine 1) i  2) n  3) nt, in each of the following case.
   a) 10% compounded annually for 2 years.
   b) 5% compounded semiannually for 4 years.
   c) 8.5% compounded quarterly for 5.5 years.
   d) 7% compounded monthly for 10 years.

31. A deposit of $5,000 earns interest at 5% per annual compounded quarterly. After two and half years, the interest rate is changed to 6% compounded monthly. How much is the account worth after six years?

32. A bank offers $10,000 five year deposit certificate at 7.5% compounded semiannually. A local credit union offers the same type of deposit certificate at 7% compounded monthly.
   a) Which investment earns more interest over the five years period?
   b) What is the difference in the amount of interest between the two offers?
   c) Explain the cause of the difference in the amount of interest between the two investments?
33. Matthew’s credit card balance was $1,300 at the end of July. Because Matthew lost his job he couldn’t pay the balance until the end of December. If the credit card company charges 1.5% per month on any unpaid balance, how much was Matthew’s credit card balance at the end of December?

34. “Compound interest is calculated not only on the original principal but also on the interest already earned”. Explain with an example.

35. Explain briefly the terms below.
   a) Compounding period
   b) Interest compounded quarterly
   c) Present Value/principal
   d) Effective interest rate.

36. Albert borrowed $7000 at 13% compounded semi-annually. He repaid $2000 after two years, and $2500 after three years. How much will he owe after five years?

37. What sum of money invested at 6% compounded quarterly will grow to $4,500 in ten years?

38. What is the present value of $5000 payable in 12 years if the current interest rate is 7.5% compounded monthly?

39. What are the differences and similarities between effective interest rate and annual percentage rate (APR)?
40. Ross has a loan of $6000 compounded quarterly for four years at 6.5%. What is the effective interest rate?

41. Marcos borrowed $420,000 to buy a house. Interest is charged at 7.2% per annum compounded monthly. How much does he owe at the end of the first month after he has made a $4000 repayment?

**U1 Answers Chapter 3- Answers to Review, Exercises and Assignments**

1. a) 12p + 9p  
   b) $780
2. a) p = 4n – 5  
   b) P= 75  
   c) P= 31
3. n = 50  
   4. $11.02  
   5. $53.67  
   6. $11.00  
   7. p +x  
   8. c= 3x
9. a) TP= 12h + 5h, b) TP= $680  
   c) TP= 12h + 5h +1.50H, h<40 and where H= overtime hours.
10. a) P=$500, b) $850  
    c) $1200  
    d) $3300.  
    11. a) $50, b)$90, c) $170  
    d) $250  
    e) $270
12. a)6t  
   b) $18  
   c) 9 hours  
   d) $4.50.
13. a) i) 380  
    ii) 655  
    iii) 710.  
15. x= $45,900, gross profit
16. ns= GP + CGS.
17. $50  
   18. a) 30 washing machines; 21 dryers  
    b) washing machines $14,400, Dryers $7,350.
19. Children tickets 212; adults tickets 53.
20. $446  
    21. x = 3  
    22. Approximately 16 students, 23. G= 39.32
24. 11200  
    25. y = 5  
    26. a) 300 widgets  
    b)$1500  
    c) $500
27. a) A=5  
    b) c= 20  
    c) k=4
28. a) B=8  
    b) x= 6  
    c) x = 3  
    d) y= 84  
    29. a) 400 T-Shirts, b) 1000 T-Shirts  
    c) $6000
30. a) 2800  
    b) 4600  
    c) $7.00  
    d) $18
31. a) $140  
    b) Approximately 35 units  
    c) $233,087  
    d)$7000.
32. $1509.91  
   33. a) Regular seats, 10,300  
    b) Front seats, 6,000  
    c) $72,000
34. 44 silk shirts, 132 tie-dyed T-shirts.
35. Elaine sold 8 subscriptions; Rob 2 subscriptions
36. Jenetta supervises 12 clerks; Edna supervises 2 clerks

37. a) n=0 b) a=\frac{4}{7}, c) x=7, d) x=7, e) x=6, f) x=14, g) a=12, h) y=6.

38. a) 2x + 4, b) 10(x -14).

39. $3125 40. 5 41. r = \frac{19}{5} 42. 2760 43. Star tire 48 units; jungle tire 72 units.

44. F= 2400 45. a) $20500 b) $11.00 c) $244,750 46. 24 ball pint pens; 12 felt-tip pens
47. 200 kids’ cards; 400 nature’s cards. 48. 26 customers
49 a) Two times a certain number decreased by 4. b) Seventeen reduced by a number multiplied by two.
50. Executive chairs 11, Secretary chairs 29.
51. a) T= 4p + 2q, b) T= 240. 52. a) $10.10 b) 12.5 hrs
53. a) V=$600, b) V=$1050 c) V=1200. 54. C= 15n + 450
55. 9 56. $62, 57. a) C=12, b) t= 79, c) x = 576, d) t= 24, e) x= \frac{5}{2}, f) a=5

g) x=11, h) x= \frac{17}{3}.

58. a) 4 +2x

b) Four times certain number increased by two.

c) 10(y -14)  d) Five times the sum of a number and 2 less 3. e) \frac{2}{3} a – 6,

f) Two times a number increased by 5, g) 3x – 50  h) Half a number increased by 8

i) \frac{3}{4} (a +12).

59. C= 0.86w + 4.49 60. 5 apartments at $625/month; 9 apartments at $900/month
61. q= 37 62. a) Rent-way, C= $200; Cheapies, C=$175; Reasonable cars, C=$250;
Drive away, C= $200; Super-Rent, C= $225. b) Cheapies
63. a) New Tunes $16.50; New Music $16.25 b) 16 songs c) New Tunes’ songs will cost less.
64 a) +1, b) +15, c) +17, d) -9, e) -12, f) +3. 65. a) -18, b) Subtraction.
66. a) 8x, b) 6x+y, c) -2a, d) 6b d) 15xy e) -2mn f) 2x + 2y g) 3t + s
67. a) y =-2, b) x = 11, c) y= 5, d) a=5  e) a=5  f) n= 6, a=5, h) n=9.
68. 500 min, 69. a) b= 140 w + 2960  b) b= $4080
70. $10 71. a) S=8g  b) 3 security guards.
72. $45. 74. 280 headlights; 720 taillights

75. a) $0.50, b) $1.30, c) $2.80, d) $3.80  
76. a) $3.75, b) $5.00  
c) $7.50  
d) $7.75  
e) $10.00

77. a) C = 675 + 2.75x  
b) i) $702.50, ii) $1500, iii) $812.50

78. 300 date books; 200 calendars.

79. a) 50 cases of cosfet; 94 cases of standard paper, b) Cosfet, $798; standard, $916.50

80. Zubes, $270000; Matt, $1080000. 81. Males 390; females, 410.

82. A, $29000; B, $31000. 83. a) -13, b) -33, c) -19

84. a) 8, b) 11, c) 12, d) 9. 85. a) 14, b) -9, c) -1.6y, d) x

86. a) +15, b) +8, c) +14.5, d) +x

87. a) $m^9$, b) $y^4x^3$, c) $s^4$, d) $a^2b^2$

88. a) 0, b) 3t, c) 12y + x, d) 25v + w. 89. 8cy

90. a) C = 2.75 + 1.57k, b) C = $38.86  
c) $11.10, d) $6.10.

91. a) 8hr 15min, b) $12, $24, $36, $48.

92. $295000; 94. 20x + 50, 95. a) $\frac{1}{16t^2}$, b) $x^8$

96. V = 6.93 97. 297.64 98. $60000$, 99. a) 6x, b) 6x  
c) 10y - 2x + 1, d) 6a + 2.

100. 2h - 6, 101. f = $\frac{3}{4}p - 50$

102. a) 34, b) 31. 103. Plusta plan (based on 200 minutes), $24; Modern plan (based on 200 minutes, $20. Modern’s plan is cheaper than Plusta’s plan.

104 a) $450, b) 1000 hot dogs, c) The 0.50 is the selling price per hot dog. The $500 is the fixed cost or set-up costs. d) 0, e) $250, f) If the team sells only 950 hot dogs, it can’t cover the set-up cost. However, if team finds ways to reduce the set-up to $475, it could sell 950 hot dogs.

105. a) 5x, b) 6x, c) -2x + 10y + 1.

106. a) 4r = 32, b) 8 + 4r = C

107. 6; 108. a) x = 1; b) $x = \frac{19}{6}$

109. C = 40 + 1.80K.
U2 Ratio Answers Chapter 5: Answers to Review, Exercises and Assignments

2. a) 1: 2, b) 21:14, c) 5 :8:13, d) 8:25, e) 1:8, f)5:4, g) 2:1: 0.7
3. a) 24:5, b) 3:25, c) 15:4:6, d) 400:1.
4. 0.17: 0.11: 0.13, 5. a)80:101, b) It means that out of every $101 she earns, she uses $80.00 to pay personal debt. 6. a)400:1. This means that each salesperson brings in $4000 and gets paid for $3000. The salespersons only bring in enough to pay for their salaries and contribute a little toward general expenses and profit.
7. a) 13:5, b) This return means that for every $13 spent $5 is saved.
8. a)       2000 2001 2002
              Multi B.  10  15  6
              All light  4  5  6
              Vim B.  5  2 1
b)Multi-Bakery did best. c) Profit ratio helps us to make comparison of the profit of a company in relation to others.
9. a) 9:20, b) It means that the bank gives out almost half of the deposits as loans.
10. $1,960. 11. Arctic Transportation gives a better deal.
12. a) 900g for 0.46 is a better buy.
       b) 1500grams of $11.51 is a better buy.
       c) 798ml is a better buy.
13. a) 191: 20,  b) 100:3,  c) 3:170,  d) 4000:3, e) 1:4
14. a) 2:1, b) For every $2 liability Western Technology can only cover half from its own resources. This is not a very good financial position for the company.
15. a) 2:3 16. a) 13 providers. b) 80 children, c) Yes it does. 17. $2.33
19. a) 1:8,  b) 2.25 cups, c) 42 litres of lemon juice.
19. a) 161 CDs, b) 48 cassettes.
20. a) $1.68, b) $4.80, c) $0.48 d) $2.40. 21. 4:0.25, 22. a) 10:13, b) 11:18, c) 105: 29.
23. a) 3:17, b) 12:7 24. a) Tanko Ltd has $2 cash for every noncash assets of $3. Supreme Ltd has $1 cash for every non-cash assets of $3. Tank Ltd. is in a much better cash position.
b) It is important because it indicates how much cash a company has in its hand. c) Supreme Ltd. because it has only $1 cash for every $3 non-cash resources.
d) $3700000.
25. 2.5. 26. 28 text messages. 27. n=700000; obtained by dividing each term by 50.
28. Manger, $30000; Clerk, $20000; Administrative assistant,$15000.
29. Muthali, $12000; Cassandra, $36000; Yen, $72000.
30. Each sales associate is paid $45000; cashier gets $30000.
32. A, $645; B, $903; C. $1032. 33. 20 34. A, $15000. 35. a) $75 b)$35.
36. 630 grams. 37. $500, 38. a) k=160, a=12, b) m= 7/3, c) b=3, f= 98.
39. $28.95. 40. a) 1:4, b) 200:1  c) 1:1000. 41. $5.60.
43. $4590, 44. 50cm. 45. a) 1:4, b) 3/10 c) ¼ , d) 18:1.
46. m=30. 47. 1:2:6; B made 2 times the profit that A made. But C made 3 times as much as B and six times as much as A.
48. $215600
U3 Percent Answers Chapter 6: Answers to Review, Exercises and Assignments

1. a) 20%, b) 20%, c) 30%  d) 6.5%  e) 30%, f) 30%, g) 20%, h) 25%.
2. 3.75%, 3. 190%.  4. a) 1%  b) For every $100 the company earns, it uses $1 for wages. c) Answers may vary but these are typical: unionization of the plant with the result that the union asked for wage increases; wage legislation might lead to increase in the wage bill; expansion of production might have led to more hiring.
5. a) $27517.24, b) $550.34. 6. a) Year 1, b) 9.09%, c) Year 1.
7. $2.61.  8. $81200  9. TV= $332.22, VCR=$310.23, computer= $2933.33, calculator= $26.31, colour printer=$250.
10. $9800, 11. Agent receives $22,800; seller/vendor receives $357200.
12. 418.46,  13. $51. 14. 25%  15. $44710.80. 16. 160%.  17. a) 9 tires, b) 1791
18. 143.14%.  19. $1500. 20. a) 4156.83, b) 77%.  21. $34.29. 22. a) $112, b) $127
23. a) $4.65, b) $6.34, c) $53.24.  24. $9000. 25. $175.  26. a) 20%, b) 6.82%, c) 11%.
27. $21600. 28. $500880.  29. Karen 19%; Mike 12.69%. 30. 36.36%. 31. Not enough because it is only $800 less than the required $1600.
32. a) 1.68% b) 0.05%.  33. 21.05%. 34. 29.58% or 30%.  35. a) 38.75%, b) 42%.
36. 15.90% or 16%.  37. $46018.52. 38. $45.60. 39. 50%.  40. a) 1/10, b) 4/5, c) 9/10 d) 1/20 e) ¼, f) ¾ g) 7/20, h) 19/50, i) 6/25 j) 3/25, k) 18/25, l) 1/2, m) 7/50
n) 1/5 o) 9/50, p) 2/5.
41.a) $51010, b) $15904. 42. a) $41127, b) 20.91% or 21%.
43. a) $120000, b) $2820, c) $300000. 44. Cost $87.88; mark up $57.12
45. Cost $15.50. 46. 3.1%  47. 89%.  48. a) 100%, b) 72%, 28%, 15%, 13%.
b) Answers vary but the following are common: reducing both the cost of sales and operating expenses; increasing net sales.
49. a) 51.52% or 52%, b) 34%.  50. $292.99. 52. 46.73%  53. $1067.  54. 1950.
55. 58.33% or 58%. 56. $31200. 57. a) $5000, b) $75000. 58. $300000.
59. $2270.  60. No, it represents 26%.  61. 60%. 62. 0.29/lb.  63. 79.92% or 80%.
64. $48.75. 65. $68.49. 66. $1.80.  68. 56 shirts. 69. 12%.  70. $1273.17. 71. 8 %
72. 51.66 73. 2600  74. 74.24 or 74%. 75. $3.84.  76. 70%  77. 10%  78. 1.25%.
79. Yes, 8.10>6.00.  80. $70. 82. 1850. 83. $11.67. 84. 5164 people.
85. 45%. 86. $12.50 a crate. 87. $15.00  88. 100%. 89. $125.00  91. No it leads to the same result. 92. 5.27%.  93. $128.  94. 4.91% or 5%. 95. a) 600 b) 33.33% or 33%, c) 100.  96. a) 0.15%, b) 6 basis points. 97. 47.06% or 47%. 98. 50%. 99. 33.33% or 33%. 100. 8.8% or 9%.

Case Study

1. a) $16.20, b) $31560, c) $31896.
2. a) $2.25, b) $2.25, c) $0.667 or $0.77.
3. a) Tent $200.00, b) $401.28, c) $82.08.
4. i) 18%, ii) $2982.35. b) i) $633.96, ii) $1804.86, d) $942.18.

U4 Answers Chapter 11- Answers to Reviews, Exercises and Assignments


21. (-2, 2)  22. k = -4  23. h = -1/2  24. c 25. (0, 4)  26. b = 2  27. d  28. a
29. y = 3x - 2  30. (6, 0)
31. a) C = 5t + 5b. $5.00  32. c  33. a) y = -5x/6 + 3 b) y = x - 1  c) y = -2x - 1  d) y = -2  34. a) y = 3x - 9  b) y = -x - 3  c) y = 3/7 + 4  d) y = 4/5x + 5 35. a) Plot the point (0, 2). Plot 5 units vertically. From the new point move 4 units horizontally to the right for the second co-ordinate point. Draw a line through the points. b) Plot the point (-2, 0). Plot -1 to the left vertically and 1 to the right horizontally. Draw a line through them.
36. a) 2/3 is the slope and (0, -2) is the y-intercept b) The slope is 1/3 and the y-intercept is (0, -1)  c) Slope is 4/5 and y-intercept is (0, -1)  d) The slope is -4 and y-intercept is (0, 2)
37. a) m = -1/6  b) m = 0  38. y = -9/4 + 2.  39. x-intercept (3, 0);
y = intercept (0, 2). Draw a line through them. 40. a) C = 60x + 300 b) 24,000  41. a) (2000, 35000), (2001, 40000), (2002, 450,000). b) Plot the co-ordinate points and draw a line through them. 42. a) The slope is 300 and the c-intercept is 1500. b) 151,500.c) 145. d) 307.50. 43. a) (0, 2), (1, 3), (2, 4). b) (0, 0), (1, 2), (2, 4).
c) (0, -1), (1, 2), (2, 5). 44. a) The ordered pairs are (0,3), (1, 4), (2,5), (5,8).
45. a) 2 c) 2400. 46. b) 2. c) y = 2x. 47. a) -2000 and it means that car decreases in value at the rate of $2000/year. b) The y-intercept is 40,000 and it means the value of that car is $40,000 at the beginning. 48. a) y = -3x + 13. b) y = 1/2x – ½.
49. m=2. 50 a) Plot the co-ordinate points given in the table. b) 1.6% c. 2004. d) 2006. 51. x-intercept (2,0); y-intercept (0,6).

52. y = 2x + 2. 53. a) 1 b) It rises. c) (0,-2). d) (2,0) e) Plot the points(0, -2) and (2, 0) and draw a line through them. 54. 4/5. 55. The slope is 0.05.

b) it means the internet provider charges 0.05 per hour. c) The C-intercept is 10, and it means that the internet provider charges a fixed amount of $10.

56. a) Hint: use the equation C=5x +25 to get at least two ordered pairs and plot them on the graph. b) $150. 57 a) The co-ordinate points are (0, 140), (15, 200), (20, 260). Plot these points and draw a line through them. b) 12. c) y=12x + 20.

58) a) Plot the given co-ordinate points. b) 0.50. 58. a) Plot the given co-ordinate points. b) 0.50. 59. y =3x -2. 60. a) True b) True c. False. 61. (10, 6)

U4 Answers (continued)  Chapter 12- Answers to Reviews, Exercises, and Assignments

1) Maya invested $23,333.33 at 5% and $16,666.67 at 8% return on investment. Comment: Suggest reason(s) why Maya would invest more money at a lower rate of return.

2) a) The two lines intersect at the point (0, -2). Hence the solution is x = 0, y = -2

b) The two lines intersect at the point (1, 2). Hence the solution is x = 1, y = 2

c) The two lines intersect at the point (2, -4). Hence the solution is x = 2, y = -4

3) The daily cost to Ricardo is $90 to hire a carpenter, and $120 to hire a painter.

4) Ray has 190 sweet pepper plants and 60 green beans plants.

5) The plane fare is $470 and accommodation is $85 per night.

6) i) : A drink cost $0.80, and a bag of potato chips cost $0.50. Therefore a drink and 8 bags of potato chips cost $4.80. So Georgina will have $0.20 change. Note: The cost of any of the other combinations is more than $5.00

7) a) Round-trip plane fare is $700.

b) Accommodation per night is $100

c) The length of stay has no effect on plane fare and accommodation per night.

8) a) x = 4, y = 1 b) x = 16, y = -15 c) x = -4, y = 2

9) a) The annual fee is $483.40. b) The green fee is $25.20

10) Let the total parking fee in $ = C, and length of parking time in hours = T.

a) North Parking Lot: C = 10 + 0.50T; South Parking Lot: C = 5 + 1.50T
b) Plot graph of two equations above on same coordinate axis (T, C).

c) The two lines meet at the point (5, 12.5). So Parking for 5 hours cost the same at both lots. *(The Charge for 5 hours parking is $12.50 at both lots)*

11) a) Mark’s operating cost may be $300 and Yan’s operating cost may be $400.

   b) \(n = 30\), \(P = 150\). If each mows 30 lawns, they make the same profit $150.

12) a) The jacket cost $143.33. b) A shirt cost $63.33 (both ‘approximately’)

13) Engine oil is $2.79 a can. Gasoline is $1.39 a litre.

14) Refer to text and from your own experience with the methods.

15. a) Let the price of pen in dollars = \(p\), and price of calculator in dollars = \(c\). Then the equation for the first situation is: \(4p + c = 54\) and the equation for the second situation is: \(2p + 2c = 72\)

   b) The price of a pen is $6 and the price of a calculator is $30.

16. Let price of donut = \(d\), and price of resin cookies = \(c\) all in dollars. Then
   
   Igor’s transaction is summarized by the equation: \(2d + 3c = 3.30\)
   
   Paul’s transaction is summarized by the equation: \(5d + 2c = 3.30\)

   Solve the equations simultaneously. Donut cost $0.75, Resin cookie $0.60.

17. This is from your perspective.

18. Number of rides for children is 8, and number of adult rides is 3.

19. a) \(x = 2.5, y = -0.5\)  
     b) infinite number of solutions: \(x = k, y = -3k -2\)

20. \(D\); \(x = 16, y = -3\).

21. The anticipated number of product \(A = a\), and the number of product \(B = b\).

   From the question we get the two equations: \(a + b = 220\) (Total number of units produced)
   \[
   \frac{a}{3} + \frac{b}{2} = 100 \quad \text{(Total machine time)}
   \]

   Find simultaneous solution of the two equations. Then the company can produce 20 units of \(A\), and 160 units of \(B\).

22. Infinite number of solutions: \(x = 7 - 4k, y = k\) for any number \(k\). Example if

   \(k = 0\), then \(x = 7, y = 0\) is a solution. If \(k = 5\), then \(x = -13, y = 5\) is a solution.

23. Ama charges $5.85 for the vase. *(She sells a Rose for $2.25)*.

24. The point of intersection of the two lines is (-1, 3). So solution is \(x = -1, y = 3\).

25. a) Infinite number of solutions. b) One solution

26. The cost per unit of hydro is $0.32. Cost per unit of gas is $0.16

27. Approximately, the Pine is $0.27 per metre, and the Redwood $0.34 per metre.

28. Approximating to 2 decimal places: 18 HDTV, and 67 Standard TVs produced
29. \( x = 0 \)

30. Stephen bought 4 CDs at $15 per CD, and 6 CDs at $20 per CD. That is Stephen bought 6 of the expensive CDs, and 4 of the cheaper CDs.

31. Ticket sales for the end of year concert amounted to $17,000

32. a) Slice of Pizza was $2  b) A can of Pepsi was $2

U4 Answers (continued) Chapter 9- Answers to Reviews, Exercises and Assignments

1. Telco.  2. 13 boxes  3. a) $4.70, b) ten rides is cheaper. c) monthly pass cheaper. d) Those who use it frequently by purchasing the monthly pass.  4. a) 10.4% b) $184  5. a) Approximately 36 times. b) $12.00

6. a) Garage A : $13.75; Garage B: $15.50 ; b) 3 hours closest  7. a) 2/5 b) 40%  8. a) 50 hours b) I assume that her hourly rate will remain unchanged.  9. 66.7%  10. 1750

11. 10 seniors 12. 30x10 13. Width should be 28 cm.

14. $2662  15. a) 7 managers  b) 60 games per year 16. $621.12  17. $202.69

18. a) 100 b) ii) 125  19. a) $166.67 b) $22.37, $15

U5 Answers Investment  Chapter 10- Answers to Reviews, Exercises and Assignments

Answers to section10.6

3. a) \( \frac{2}{3} \)  b) \( \frac{1}{4} \)  c) \( \frac{1}{200} \)  d) \( \frac{1}{5} \)  4. a) 0.23 b) 2.50 c) 0.00003 d) 0.0001 e) 0.045

5. a) 45% b) 125% c) 0.7% d) 4%  6. 24%  7. a) 16% b) 0.025  8. a) 3.4% b) 0.0305

9. a) 0.5% b) $135  c) $675  10. a) \( \frac{1}{40} > \frac{1}{4} \% \)  b) \( \frac{1}{5} \% < 0.04 \)  11. a) $137.50  b) 36%

12. a) $37.50  b) $750  13. a) $540  b) $18  c) $11.84  14. a) 1200  b) 80%  16. 6%

17. 6 months  18. 3 months  19. 20.54%  20. a) $6.36  b) $221.36  22. $1 604.38  25. 18.1%

Convert all to annual rate.

26. a) \( \frac{2}{3} \)  b) \( \frac{10}{3} \) or \( 3 \frac{1}{3} \)  c) \( \frac{7}{3} \) or \( 2 \frac{1}{3} \)  d) \( \frac{3}{2} \) or \( 1 \frac{1}{2} \)  27. $19 250  28. a) $2 165.63  b) $7 665.63

29. $33 500  30. $3 125  31. 8 months  32. 18.5%  33. 2 years  34. $25  35. $13.75

36. About 7 months  37. A) $5.70  b) B is better when amount borrowed is >$923.08

38. $31 666.67  39. $233.49  40. $6 041.67  41. $10 615  42. b

43. The number of months she must pay $80.  44. $626.40
U5 Answers (continued)   Answers to section 10.14

2.   a) $5 891.20   b) $5 896.93   c) $5 898.24   d) $5 898.88 (for 365 days in a year)
3. a is $625, b is $625.90, and c is $623.03, so b is a better way to invest.
4.   a) $756   b) $832.14   5. a) 10 years   b) 8 years (interest is annual)
6. For 1 year, the interest from Option 1 is $30.34, and Option 2 is $30.00. So Option 2 makes more money.
7. a) $28 142.11   b) $7 642.11   9. $2 643.41   10. $15 453.30   11. $16 803.20
12. The present value of $580 in a year’s time is $514.72 which is more than $500. So based on book value $580 in a year’s time is a better deal than $500 today. The future $580 could be sold for more than $500 today. 13. $5 135.68 (assume each year is 365 days).
14. b) $702.50   c) $1 720.24   d) $1 620.99
15. a) $10 068.09   b) $5 068.09   16. a) $6 100.14   b) 7.55% per year
18. Maturity value is $3 516.78, Interest is $516.78. 19. $15 000 today. Refer to 12.
21. a) $2 547.88   b) 8.49%   22. $6 800   23. a) $2 552.56   b) About 14 years 3 months
24. $1 400.47   26. a) $16 866.03   b) $19 729.53   c) $28 446.30   d) $47 977.61
28. a) 40   b) 2%
c) (1.02)^40   d) 2.20804   29. a) 1.25%   b) 0.625%   c) 2.85%   d) 8%   e) $\frac{2}{11}$ or 0.027%
31. $6 980.63   32. a) Bank’s investment   b) $274.19   33. $1 400.47   36. $7 721.68   37. $2 480.68
38. $2 038.55   40.7.36% per year   41. $418 520   42. 19.56%   43. a) $34 192.17   b) 4.04%   c) 12.68%
44. $1 715.22   45. $5.58   46. a) $2 252.99   b) $2 859.00   47. 3 763 000.