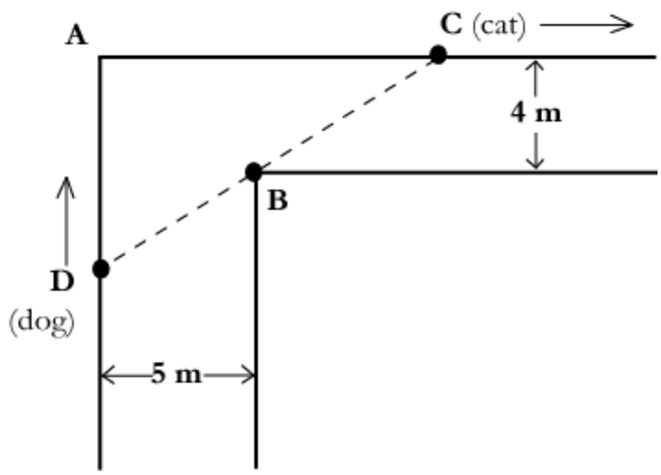
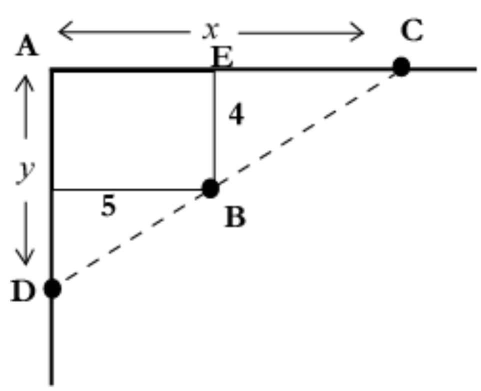
Example 1(taken from notes in Advanced Placement Calculus Textbook)

Question: In the diagram shown the corridors are 4 m and 5 m wide. A cat C is moving to the right at 10 m/sec. At the instant when a dog D is 12 metres from A, how fast must the dog run in order to keep the cat in sight?



Answer: Let $AD = y$ and $AC = x$. We wish to find $\frac{dy}{dt}$ when $y = 12$ and $\frac{dx}{dt} = 10$.



In the diagram

$\Delta DAC \sim \Delta BEC$

$$\therefore \quad \frac{DA}{BE} = \frac{AC}{EC}$$

$$\therefore \quad \frac{y}{4} = \frac{x}{x-5}.$$

$$\therefore \quad y = \frac{4x}{x-5} \quad (1)$$

Differentiate (1) with respect to time:

$$\frac{dy}{dt} = \frac{(x-5)4\frac{dx}{dt} - 4x\frac{dx}{dt}}{(x-5)^2}$$

At the special instant when $y = 12$, $x = 7.5$ from (1)

and $\frac{dx}{dt} = 10$. We have:

$$\frac{dy}{dt} = \frac{(7.5-5)4(10) - 4(7.5)(10)}{((7.5)-5)^2}$$

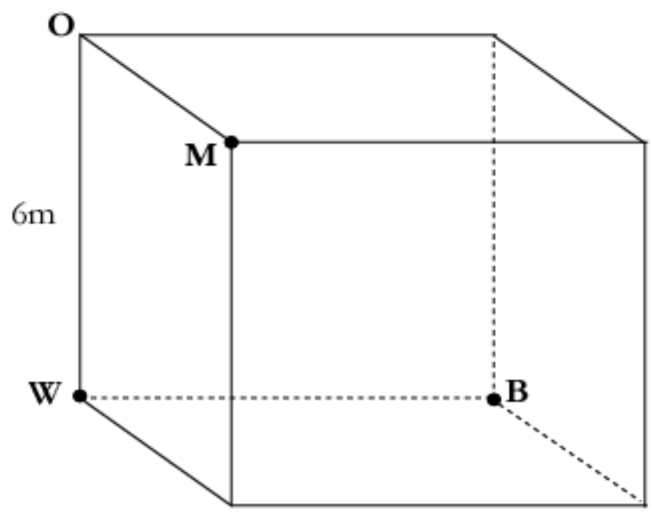
$$= -32 \text{ m/sec}$$

∴ The dog must move at 32 m/sec to keep the cat in sight. (The dog is going to be disappointed.)

<u>Example</u> (in 3-dimensional space) again from *Advanced Placement Calculus Textbook*

Question: A man is running over a bridge at a rate of 5 metres per second while a boat passes under the bridge and immediately below him at a rate of 1 metre per second. The boat's course is at right angles to the man's and 6 metres below it. How fast is the distance the between the man and the boat separating 2 seconds later?

Answer:



In the diagram let M be the position of the man, let B be the position of the boat. OM represents the distance the man runs.

WB represents the distance the boat goes along the river.

OW is 6 m i.e. the height of the bridge above the water.

Let OM be x, let WB be y and let MB be s, the distance separating the man and the boat.

$$s^2 = x^2 + 6^2 + y^2 \qquad (1)$$

After two seconds, $x = 10$ m, $y = 2$ m, and hence $s = \sqrt{140}$.

Differentiate (1) with respect to time and cancel by 2.

Then $s\dfrac{ds}{dt} = x\dfrac{dx}{dt} + y\dfrac{dy}{dt}$

After 2 seconds we have

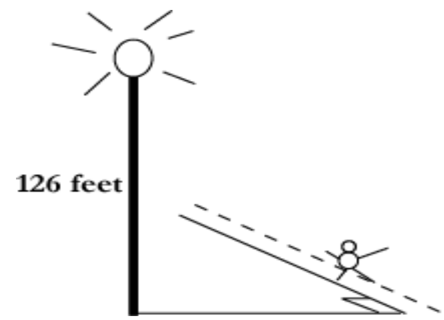
$$\sqrt{140}\frac{ds}{dt} = 10(5) + 2(1)$$

$\therefore \qquad \frac{ds}{dt} = \frac{52}{\sqrt{140}} = 4.395 \text{ (approx.)}$

$\therefore$ The distance between the man and the boat is increasing by 4.395 m/sec.

Example 3: Another 3D problem- forgive the diagram! (Again taken from *Advanced Placement Calculus Textbook*)

A man 6 feet tall walks along a walkway which is 30 feet from the base of a lamp which is 126 feet tall. The man walks at a constant rate of 3 feet per second. How fast is the length of his shadow changing when he is 40 feet along the walkway past the closest point to the lamp?



Answer: 0.12 ft/sec

Examples 4, 5 (Same source- my textbook)
In the first example, it is neat because the solution depends upon whether the shadow is cast on the horizontal or vertical.
In the second example, assume that the rope is taught due to the weight of the car.

A light is at the top of a pole 80 feet high. A ball is dropped at the same height from a point 20 feet away from the light. A wall 80 feet high, 60 feet away from the light is built. Assuming the ball falls according to the Newtonian Law $s = 16t^2$ where s is the distance in feet and t is the time in seconds, find:

a) how fast the shadow of the ball is moving on the wall after 1 second.

b) how fast the shadow is moving along the ground after 2 seconds.

Answers a) 96 b) 25

Two poles are 24 metres and 30 metres high and 20 metres apart. A slack wire joins the tops of the poles and is 32 metres long. A cable car is moving along the wire at 5 metres per second away from the shorter pole. When the car is 12 metres horizontally from the shorter pole, it is 15 metres high and the length of the wire to the shorter pole is 15 metres. At this instant, find:

a) how fast the car is moving horizontally.

b) how fast the car is moving vertically.

Answer: a) 7.5 b) $\frac{5}{3}$