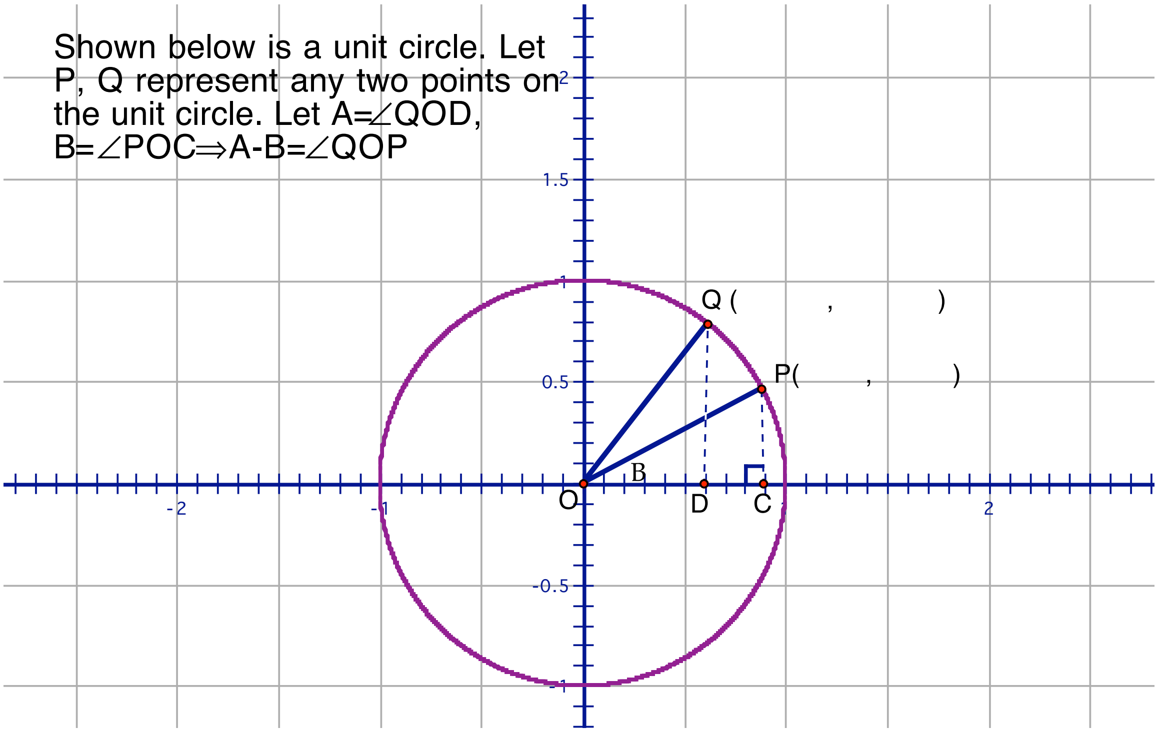
Sum/Difference, Double Angle Trigonometric Formualae

Using the diagram below, prove the identity: $\cos(A-B)=\cos A\cos B+\sin A\sin B$



Proofs for related sum/difference, double angle formulae:

Section B: Long Answer – Full Solutions Required

1. Prove the identity: $\dfrac{\tan^2\theta - 1}{\tan^2\theta + 1} = \cos 2\theta$

2. $\dfrac{\csc A}{\cot A + \tan A} = \cos A$

3. Solve for θ in the interval $0 \le \theta \le 2\pi$:

a) $2\cos^2\theta + \cos\theta = 1$

b) $\sin 2\theta = 3\cos^2\theta$

c) $2\tan^2\theta + \dfrac{3}{\cos^2\theta} = 8$

4. a) On the same set of axes graph $y = \tan x$ and $y = \sin 2x$ over the interval $0 \le x \le 2\pi$. Circle the points of intersection.

b) Find the points of intersections you circled algebraically.

5. Given that $\frac{\pi}{2} \le A \le \pi$ and $\pi \le B \le \frac{3\pi}{2}$ and that $\sin 2A = \frac{3}{5}$ and $\cot B = \frac{5}{12}$, find:

a) $\sin A$

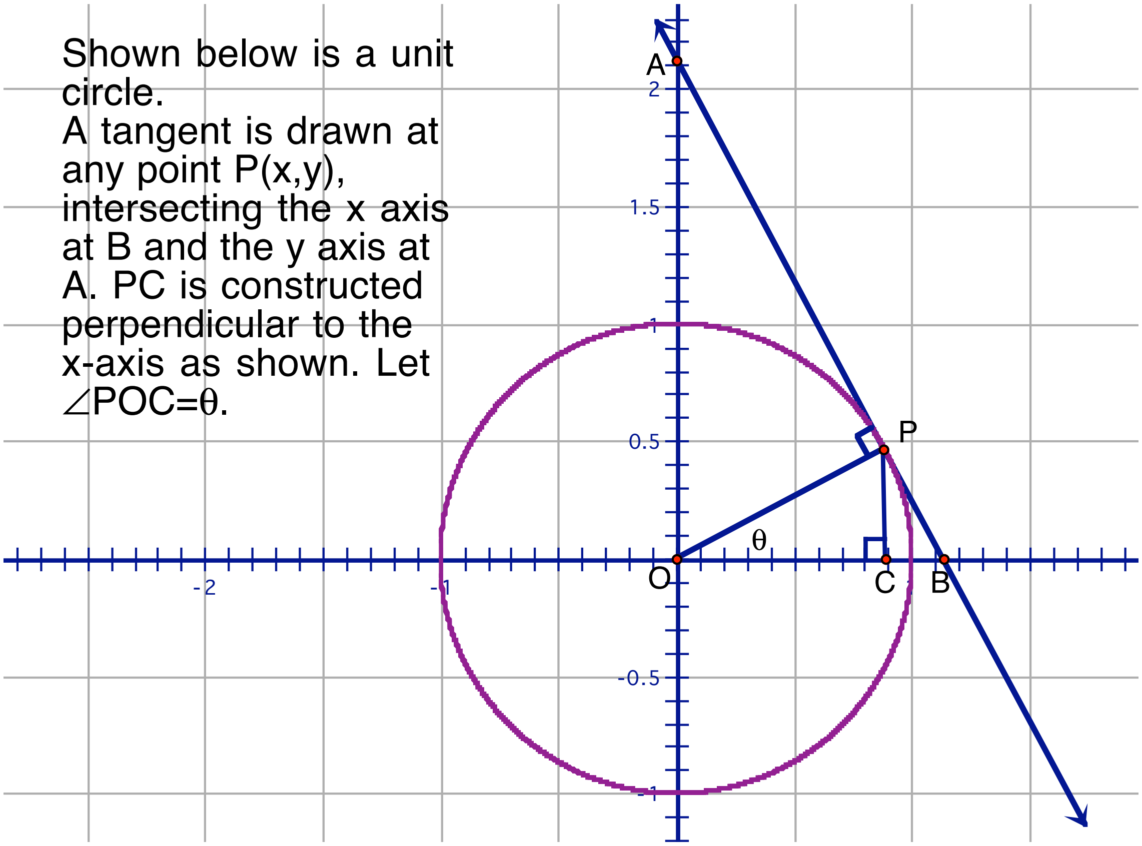
b) $\cos 2B$

c) $\tan\left(B + \frac{\pi}{4}\right)$.

Use your answer to determine whether B is greater than $\frac{5\pi}{4}$

10. Find $\lim_{\theta \to 0} \frac{\sin \theta}{\theta}$ using a geometric proof

Shown below is a unit circle.
A tangent is drawn at any point P(x,y), intersecting the x axis at B and the y axis at A. PC is constructed perpendicular to the x-axis as shown. Let $\angle POC = \theta$.



Problems- Identities, Equations- No calculators! (except for 3c)

1. If $\sin\theta = \frac{2}{3}$ and θ is acute, find the value of a) $\sin 2\theta$ and b) $\sin 4\theta$.

2. Find $\tan x$ given that the value of $\tan\left(x - \frac{3\pi}{4}\right) = 2$

3. Solve each of the following in the interval $[0, 2\pi]$.

a) $\sin^2 x + \cos^2 x = \cos x$

b) $\sin^2 x \cos^2 x = \frac{3}{16}$

c) $\cos x + \tan x = 0$

d) $\cos 2x = \sin(x + \frac{3\pi}{2})$

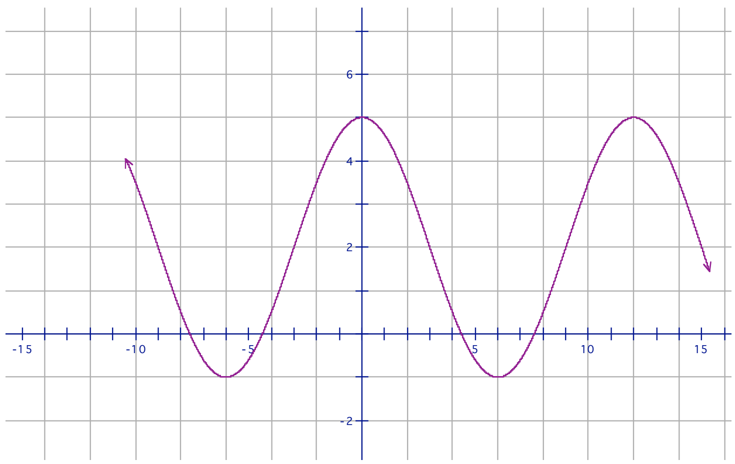
e) $\tan(2x) = \frac{1}{1 + \tan x}$ (you may use your calculator for this near the end)

4. Prove each of the following identities:

a) $\frac{\sin 2x}{1 + \cos 2x} = \tan x$

b) 2csc2x=secxcscx

5 Predict an equation for the following graph:
Note: the first **minimum** value of x>0 is at the point $(6, -1)$



6. At the ocean, it is known that the tide follows a trigonometric path. At high tide, the water comes in to a point 1 metre from where I placed a flag. At low tide, the water comes in to a point 11 metres from the same flag. The time it takes from to get from high tide to low tide is 5 hours. It is now midnight and it is high tide. (Note: low tide=max and high tide=min in this case)

a) Plot the motion for two complete cycles below:

b) State a possible equation for this motion.

c) We want to wake up and go to the beach when we can set up our towels at a time between 10 am and 2 pm the next day when the water will be 4 metres from our flag. At what time will this be? Explain what you did, even if you used your graphing calculator to find the answer.