

Exponents and Logarithms Review

1. Solving exponential equations:

Solve :

$$a) 8^x = 4^{-x-3}$$

$$b) 3^{x+1} + 9^x = 18$$

$$c) 3x^{\frac{2}{3}} = \frac{1}{3}$$

2. Recall: Terminology of Logarithms

If $10^x = 100$ then of course, $x = \underline{\quad}$.

However, it is equivalent to write the original statement as $x = \log_{10} 100$.

In general, we can define this as if $b^x = a$ then $x = \log_b a$

Key Properties of logarithms

$$a) \log_a b + \log_a c = \underline{\hspace{2cm}}$$

$$b) \log_a b - \log_a c = \underline{\hspace{2cm}}$$

$$c) n \log_a b = \underline{\hspace{2cm}}$$

$$d) \log_a b = \frac{\log_c b}{\log_c a}$$

used to change the base when needed

$$e) a^{\log_a b} = \underline{\hspace{2cm}}$$

Proofs:

3. Simplify the following using the properties listed above:

a) $\log(20)+\log(5)=$ b) $\log_7 3 - \log_7 21 =$ c) $\log_3 9^{17} =$

d) $\log_4(x^2 - 4) - 2\log_4(x + 2)$ e) $\log_8 \sqrt{8x}$
=

4. Solve exponential and logarithmic equations:

a) $2^x = 100$

b) $2\log(x - 1) - \log(x + 8) = -1$

5. The value of the constant e :

You are given \$1 and are given an basic rate of interest of 100% per annum (very generous). You may compound this 100% rate as frequently as you wish. Find the limit of how much money you could end up with after one year.

6. The use of $e^x, \ln x$ in equations:

Solve:

a) $e^{2x} = 40$

b) $3 \ln x + 1 = -2$

c) Find the intersection of the graphs of $y = e^x + 3$
 $y = e^{2x} + 1$ algebraically.

d) Solve for x : $\frac{1}{\ln x} - 6 \ln x = 1$

e) State the domain, range of $y = x \ln x$

Exponents and Logarithms Problems

1. Solve for x :

a) $2^{-x+1} = 4(8^x)$

b) $e^{2x} = 20$

c) $e^{2x} = e^x + 2$

d) $\log x = 2$

e) $\ln x = 2$

f) $\ln(\ln x) = 2$

g) $10^{\log 20} = x$

h) $\log(9x+1) - \log(x-1) = 1$

i) $\ln x + \ln x^2 = 6$

2. Find the points of intersection of the following algebraically. You may check with your calculator.

a) $y = 2^x + 4^x$
 $y = 2^{x+1} - 4^{x+1}$

b) $y = \ln x$ and $y = 2 - 3 \ln x$

3. Solve each of the following:

a) $4^x + 4^{x-\frac{1}{2}} = 45$

b) $3^{p+1} + 3^p = 70$

c) $(3^m)(5^{m+1}) = 12^{2m-1}$

4. Find any intersection points of the graphs of

$y = 2 \log_3(9x)$ and $y = \log_3(x+8) + 2$.

5. Solve: $\log_x 4 + \log_8 \sqrt{x} = \frac{4}{3}$

6. Solve for x in the interval $[0, 2\pi]$:

a) $\log_2 \sin x + \log_2 \cos x = \frac{-3}{2}$

b) $\log_2 \cot x - 2 \log_4 \csc 2x = \log_2 \cos x$

7. Find the domain, range of $y = \frac{x}{\ln x}$
8. Consider the functions $f(x) = \ln(\sin^2 x)$.
- a) Find the domain, range of $f(x)$
- b) Let $g(x) = 2 \ln(\sin x)$. Explain why $f(x) \neq g(x)$.

Answers

1. a) $x = -1/4$ b) $\frac{\ln 20}{2}$ c) $\ln 2$ d) 100 e) e^2 f) $e^{(e^2)}$ g) 20 h) 11 i) e^2
2. a) $x = \frac{\ln(1/5)}{\ln 2}$ or $\log_2\left(\frac{1}{5}\right)$, $y = \frac{6}{25}$ b) $x = e^{1/2}$ and $y = 1/2$
3. a) 2.45 b) $\frac{\ln(17.5)}{\ln 3} \doteq 2.61$ c) 1.81 4. (1,4) 5. 4,64
6. a) $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$ b) $\frac{\pi}{3}, \frac{5\pi}{3}$
7. Domain = $\{x \in \mathbb{R} : x > 0, x \neq 1\}$, Range = $\{y \in \mathbb{R} : y < 0 \text{ or } y \geq e\}$
8. Domain = $\{x \in \mathbb{R}, x \neq k\pi, k \in I\}$, Range = $\{y \in \mathbb{R} : y \leq 0\}$.
- b) Different domain as $g(x)$ is only valid when $\sin x \geq 0$