**AP Test- Trig, Exponential Functions, Integration   Name: ______________**

**Section A- No Calculator- Multiple Choice   /19   (est. time 15 minutes)**

1. If $f(x) = (\ln x)^2$, then $f'(2) =$ (3 marks)

$a) -2\ln 2$   $b) -\ln 2$   $c) \ln 2$   $d) 2\ln 2$   $e) \frac{1}{3}(\ln 2)^3$

2. $\lim_{x \to 0} \frac{\sin x}{xe^x}$ = (2 marks)

a) 0   b) 1   c) -1   d) 2   e) e

3. $\int_0^{\frac{\pi}{6}} 2\sin 2x dx =$ (2 marks)

a) $\frac{\sqrt{3}}{2}$   b) 1   c) $\frac{1}{2}$   d) 0   e) -1

4. $f(x) = \frac{x}{\tan x}$, then $f'(\frac{\pi}{4}) =$ (2 marks)

$a) 1 - \frac{\pi}{2}$   $b) \frac{\pi}{2}$   $c) \frac{\pi}{2} - 1$   $d) 4$   $e) \frac{\pi}{4}$

5. A population, in millions, at any time t is given by the formula

$P(t) = \dfrac{20(1-e^{-2t})}{5-e^{-t}}$ . The $\lim_{t\to\infty} P(t)$, in millions, is: (1mark)

a) 20 b) 4 c) 5 d) infinite e) 0

6. The equation of the tangent to $\ln(x+y) - e^{-y} = -x^3 + 3y$ at the point P(1,0) is: (3 marks)

a) y=x b) y = 2x – 2 c) y = x – 1 d) y = -x + 1 e) y = - 2x + 2

7. The slope of the tangent to the curve $y = (x+e^x)^{\cos 2x}$ at $x = 0$ is : (3 marks)

a) -2 b) -1 c) 0 d) 1 e) 2

8. $\lim_{x\to 0}[1-\ln(x+1)]^{\frac{1}{x}} =$ (3 marks)

a) $\dfrac{1}{e}$ b) 0 c) $\sqrt{e}$ d) 1 e) $e$

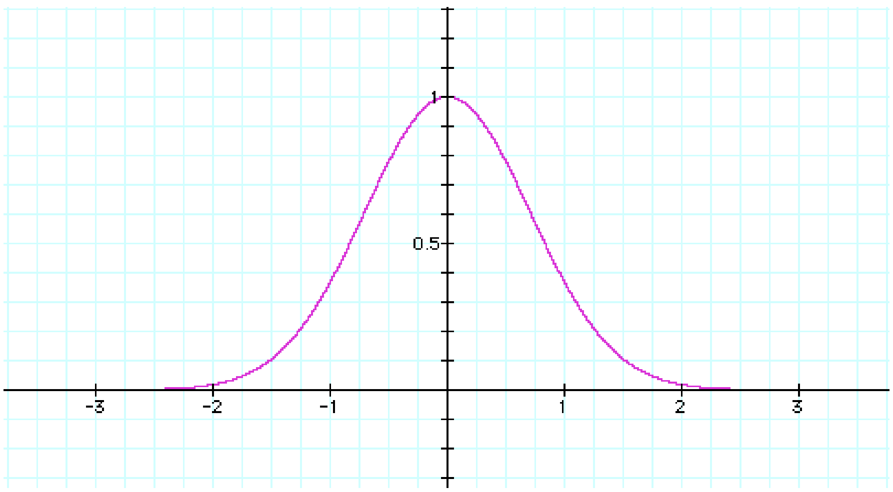
## Section B- Full Solutions Required /38

Instructions: Calculators may be used. Justification is only required where asked for. Answer on the lined paper provided.

1. Find each of the following (you need not show any working, if it is not necessary) (7 marks)

a) $\int \frac{4x^2-2x}{x+1}dx$  b) $\int x^2(x^3-2)^5\,dx$  c) $\int \frac{\cos 3x}{(1-\sin 3x)}dx$

2. A rectangle is made with two vertices on the curve $y=e^{-x^2}$ and two vertices on the x-axis . Find, using algebra, the value of the vertices on the x axis that maximize the area of the rectangle. (6 marks)



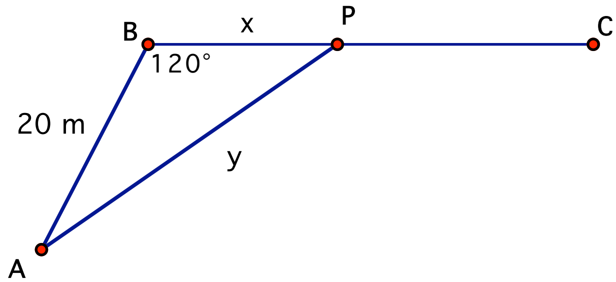

3. a) The tangent line to the graph of $f(x)=e^{2-x}$ at the point P(1,e) intersects both the x and the y axes. Find the area of the triangle formed by this tangent and the coordinate axes. (4 marks)

b) The graph of $g(x)=a\ln x+b$ intersects $f(x)=e^{2-x}$ at the point P(1,e) at right angles. Find the values of constants $a$ and $b$. (4 marks)

4. The position of a point on a number line is given by the formula
$s(t)=2(t-1)^2+\ln(4t+1)$ where $t\geq 0$. (t in seconds, s in metres)
Find: (8 marks)
a) the velocity of the particle at t= 1 second.
b) the time(s) at which the particle is stopped (justify using algebra)
c) the total distance travelled in the first 5 seconds (round to two decimals).

**Turn page over for #5**

5. An observer at A is watching a cyclist move in a straight line from a point B towards another point C at a constant speed of $10 m/s$ as shown in the diagram. The initial distance between the observer and the cyclist is 20m and angle ABC is $120^0$. At any time later, the cyclist is now at P. (9 marks)




a) Show that $y^2 = x^2 + 20x + 400$

b) How far is the cyclist from the observer 6 seconds after the cyclist leaves A?

c) At what rate is the distance between the observer and the cyclist increasing at this time?

d) Find the rate at which angle BPA is changing at this instant, in radians per second, rounded to two decimals.

## Section B- Full Solutions Required /38

Instructions: Calculators may be used. Justification is only required where asked for. Answer on the lined paper provided.

1. Find: (9 marks)

a) $\int \frac{5x^2 - 2x}{\sqrt{x}} dx$ b) $\int \cos 3x (1 + \sin 3x)^4 dx$ c) $\int \frac{2e^x - 4x}{e^x - x^2} dx$

and now, using a calculator: d) $\int_1^3 \left(5 \ln x + e^{x^2}\right) dx$

2. A missile rises vertically from a point on the ground 7.5 km from a radar station. If, at the instant when it is 3.8 km high, the missile is rising at the rate of 16.5 km per minute, what is the rate of change in the missile's angle of elevation from the radar station at this instant? Round answer to two decimal places. (6 marks)

3. a) Determine, algebraically, the smallest positive value of where the **slope** of $f(x) = 8\cos x$ and $g(x) = \cot x + k$ are equal. (4 marks)

b) Hence, find the exact value of the constant $k$ such that these graphs will be tangent to each other at this value of $x$. (2 marks)

4. The height, in metres, of an object at time t seconds is given by the formula $h(t) = 5\ln(t^2 + 1) - \frac{1}{10}t^2$. Find, rounding answers to two decimals: (7 marks)

a) The maximum height of the object (justify algebraically).

b) The time it takes for the object to land, using a calculator to justify

c) The total distance travelled until it lands.

See next page for #5

5. The velocity of a Canadian swimmer during a race is modelled by the formula $v = \frac{1}{14}\sqrt{t} - 2e^{-t} + 2$, where t is in metres and v in metres/second

(10 marks)

a) Find the formula for the position of the Canadian swimmer at any time t, assuming her position at the start is zero.

b) Use your calculator to determine how long it takes for the Canadian to finish the 100 metre freestyle race. (round to the nearest one-hundredth of a second)

c) The velocity of an American swimmer in the same race is modelled by the formula $v = \sin t + e^{t/40} + 0.5$. Find the formula for the position of the American swimmer at any time t.

d) It is a very close race. Use your calculator to determine who wins the race.

e) Use your calculator to find the maximum lead of the Canadian swimmer during the race to the nearest tenth of a metre.