

AP Test –Apps of the Derivative, including Trigonometry Name:Kendra
Section A: Total **20** marks. No calculators. Show your work.

1. $f(x) = \frac{x - \frac{\pi}{4}}{\tan x}$, then $f'(\frac{\pi}{4}) =$ (3)

- a) $\frac{1}{2}$ b) 0 c) -1 d) 2 e) 1

2. The slope of the tangent to $-\cos y + \sin 3x + 2 \tan(y + 3x) = -1$ at the point $(-\frac{\pi}{6}, \frac{\pi}{2})$ is: (4)

- a) -3 b) -2 c) -1 d) 2 e) 3

3. $\lim_{x \rightarrow \frac{\pi}{3}} \frac{2 \sec x - 4}{\sqrt{3} \tan x - 3} =$ (3)

- a) $\frac{-1}{3}$ b) 1 c) -1 d) -2 e) $\sqrt{3}$

4. Given that the velocity of a ball is given by the formula $v = 10t - \cos(\pi t)$ where v is in m/s, then the average acceleration in the interval $[0,2]$, in m / s^2 is: (2)

- a) 12 b) 10 c) 4 d) 2 e) 0

5. A rectangle changes shape such that its length increases at a constant rate of 2 m/s and its width decreases accordingly in order to maintain a constant area of 50 square metres at all times. At the instant when the length is 10 metres, the width is decreasing at a rate of: (4)

- a) -1 b) -2 c) -3 d) -4 e) -5

6. A paper cup in the shape of a cone is being filled with water at a rate of $24\pi \text{ cm}^3 / \text{s}$. The height of the cone is 30 cm, while the radius is 10 cm. (4)
Find the change in the radius of the surface of water in the cup when the depth of the water in the paper cup is 6 cm. ($V_{\text{cone}} = \frac{1}{3}\pi r^2 h$)

- a) 0.5 cm/s b) 1 cm/s c) 1.5 cm/s d) 2 cm/s e) 2.5 cm/s

Section B- Calculators may be used. Please justify algebraically as indicated.

Write solutions on lined paper.

$\overline{26}$

7. Assume the angle of elevation of the sun changes at a constant rate of 18 degrees per hour. The sun rises at 6 a.m. Find the rate at which the shadow made by a 10 m pole changes at 8 a.m. (6)

8. The height of an object thrown downward from an initial altitude of 200 m is $h(t) = 200 - 12t - 5t^2$. The object is being tracked by a searchlight 100 m from where the object will hit the ground. How fast is the angle of elevation of the searchlight changing after 3 s? (6)

8. a) Determine the exact value of x at which the graphs of

$$y = -4 \cos^2 x \text{ and } y = 4 \sin x + c \text{ have the same slope in the interval } 0 < x < \frac{\pi}{2} \quad (4)$$

b) Find the exact value of c such that the graphs are tangent to one another in the interval $0 < x < \frac{\pi}{2}$. (4)

10. A sign board 45 feet high stands at the top of a cliff 86 feet high. How far from the foot of a cliff should a man stand in order for the sign to subtend the largest possible angle at his eyes which are placed 6 feet above the ground? (8)